Square tessellation for stochastic connected *k*-coverage in planar wireless sensor networks

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***Abstract*— In this paper, we focus on the problem of connected *k*-coverage in planar wireless sensor networks (PWSNs), where every point in a field of interest (FoI) is covered by at least *k* sensors simultaneously, while all the participating sensors are mutually connected, where *k* > 1. To this end, we develop a global framework using a square tessellation that considers both deterministic and stochastic sensing models. Initially, we tessellate a planar FoI into adjacent and congruent square tiles. In each tile of this tessellation, we construct a cusp-square area for sensor placement to achieve *k*-coverage. Based on this cusp-squared square tile configuration, we compute the minimum sensor density that is required for deterministic and stochastic *k*-coverage in PWSNs. Then, we establish the necessary relationship that should exist between the sensing and communication ranges of the sensors to maintain network connectivity in *k*-covered PWSNs. Finally, we propose our stochastic *k*-coverage protocol for sensor scheduling and substantiate our theoretical analysis with simulation results.**

Keywords—Planar wireless sensor networks, connected k-coverage, stochastic sensing, sensor density, square tessellation.

I. Introduction

One of the important research problems in PWSNs is sensor scheduling with the primary goal of achieving reliable coverage of a FoI. There are various sensing applications, such as intruder detection and tracking, which require that each point in a FoI be sensed by at least one sensor. Specifically, *k*-coverage is an appealing solution, where every point in a FoI is sensed by at least *k* sensors, where $k \geq 1$. Also, network connectivity should be maintained between the sensors so the data collected by the sensors is successfully sent to the sink for further processing and analysis. Thus, we attempt to solve the connected *k*-coverage problem in PWSNs, where *k*-coverage along with network connectivity are ensured using a minimum number of sensors.

This paper is an extension of our previous work [20], where we addressed this problem of connected *k*-coverage in PWSNs using a square tessellation-based approach using deterministic sensing model. Here, we are interested in a realistic sensing model, *i.e.*, stochastic sensing model, where a sensor covers a point in a FoI with some probability.

A. Problem Statement

We want to investigate the connected *k*-coverage problem in PWSNs by addressing the following four major inter-related questions:

— **Q1**: What is the optimal way of placing the sensors for attaining *k*-coverage of a FoI, using a minimum number of sensors, where $k \geq 1$ is the degree of coverage?

— **Q2**: What is the minimum planar sensor density (*i.e.,* number of sensors per unit area) for achieving *k*-coverage of a FoI, using the sensor placement strategy determined in Q1?

— **Q3**: What is the relationship between the sensors' sensing and communication ranges for maintaining network connectivity using the above sensor placement strategy?

— **Q4**: What is the best way to select and schedule the sensors to *k*-cover a FoI using a deployed sensor density that is almost near the one obtained from Q2?

B. Contributions and Organization

Our contributions in this paper are summarized as follows:

— We tessellate a FoI using square tiles whose dimensions are proportional to the sensing radius of the sensors.
— We construct a cusp-square area in each square tile and deploy a minimum number of sensors in each cusp-square area for attaining *k*-coverage of a FoI, where $k > 1$.
— We compute the sensor density for the above proposed sensor placement strategy for *k*-coverage, where $k > 1$.
— We compute the necessary relationship between the sensors' sensing and communication ranges for the above proposed sensor placement strategy to achieve connected *k*-coverage.
— We generalize our solution to the connected *k*-coverage problem using a stochastic sensing model, where the sensing range of the sensors has an irregular shape.
— We propose a sensor selection protocol, where the sensors are scheduled to *k*-cover a FoI, while optimizing the overall energy consumption so as to extend the network lifetime.
— We substantiate our theoretical analysis using simulation results. We observe a close-to-perfect match between them.

The rest of the paper is organized as follows. In Section II, we introduce our models. Section III provides a review of related work. In Section IV, we solve the connected *k*-coverage problem using both sensing models and present our stochastic connected *k*-coverage protocol. In Section V, we assess it by comparing the simulation and theoretical results. Also, we compare it with existing stochastic connected *k*-coverage protocols, SCP_k [5] and RCH_k [8]. Finally, in Section VI, we conclude our paper.

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II. Models

A. Sensing Model

We assume that the radii of the sensing and communication ranges as r_s and r_c, respectively, and consider both deterministic and stochastic sensing models to solve the connected *k*-coverage problem. In a deterministic sensing model, a point *P* is said to be covered by the sensor *s* iff the Euclidean distance $\delta(P, s)$ between *P* and *s* is less than or equal to r_s. This model does not consider the uncertainty of sensor reading capabilities. Hence, considering signal attenuation and noisy sensor readings, we use a stochastic sensing model that considers the coverage $Cov(P, s)$ as the probability of detection $p(P, s)$, as in (1).

$$p(P,s) = \begin{cases} e^{-\beta\delta(P,s)^{\alpha}} & if\ \delta(P,s) \leq r_s \\ 0 & otherwise \end{cases} \quad (1)$$

where β represents the physical characteristics of the sensor's sensing unit, and $\alpha \in [2, 4]$ is the path-loss exponent.

B. Energy Model

We consider the energy model [2] that computes the energy consumed due to data transmission and reception by the sensors:

$$E_t(d) = b \times (\varepsilon d^{\alpha} + E_e) \quad (2)$$

$$E_r = b \times E_e \quad (3)$$

where $E_t(d)$ is the energy consumed by sensor *s* while transmitting a message of *b* bits over a distance *d*, E_r is the energy consumed by sensor *s* while receiving a message of *b* bits, E_e is the electronical energy, ε is the transmitter amplifier in the free-space (ε_{fs}) or multi-path (ε_{mp}) model, and α is the path-loss exponent. Using the energy model proposed by Ye *et al.* [3], a sensor consumes 0.012 J in idle mode, 0.0003 J in sleep mode, and a randomly value in [0.008, 0.012] J/m [4] when moving.

III. Related Work

Ammari [5, 6] solved the *k*-coverage problem using Reuleaux triangle-based tessellation. Yu *et al.* [9] proposed stochastic *k*-coverage protocol $ISCP_k$ which models the sensing range of sensor as four regular pentagons in PWSNs, by placing *k*-1 sensors in central areas of these four regular pentagons. Sun *et al.* [10] developed a *k*-coverage algorithm based on the optimization node deployment process. Unlike from previous research [5, 6], in [7] Ammari has worked on regular hexagon-based tessellation and constructed a generic irregular hexagon that can be laid over the regular hexagon tessellation for increasing *k*-coverage area and minimizing the number of sensors being used. Abbasi *et al.* [17] suggested a method for coverage control in continuous and potentially long regions and passages, where optimal coverage is ensured by a group of autonomous mobile sensors which move within the boundaries of the regions/passages. Similar to [12, 14, 16], Harizan and Kuila [18] addressed the *k*-coverage problem in PWSNs using heuristic and nature-inspired algorithms. Natarajan and Parthiban [19] used shuffled frog leaping Nelder-Mead algorithm, for optimal node placement to achieve *k*-coverage of target locations in FoI. Similar to [7], In [8] Ammari worked on regular hexagon-based tessellation and proposed stochastic protocol RCH_k which uses sliced hexagons for sensor placement, unlike the irregular hexagon [7]. Similar to Ammari [5, 6], Yu *et al.* [11] has used Reuleaux triangle-based tessellation and constructed coverage contribution area (CCA) for sensor placement. Krishnan *et al.* [12] has studied the performance of ten different approaches for achieving *k*-coverage in PWSNs. Chenait *et al.* [13] has proposed SRA-Per and SRA-SP protocols based on sector redundancy determination algorithm. The latter determines the redundant sensors that are not required for the *k*-coverage process by slicing the sensing range of the sensors using predefined sector angle. Elhoseny *et al.* [14] suggested a genetic algorithm-based approach for the *k*-coverage of specific target locations in a FoI, while maximizing the network lifetime. Hoyingcharoen and Teerapabkajorndet [15] evaluated the expected sensing probability at any given location as well as the expected level of connectivity to the sink for any sensor that is unable to transfer data directly to the sink. Naik and Shetty [16] exploited the DE algorithm for estimating the optimal candidate locations in a FoI for sensor deployment in order to achieve the required *k*-coverage of specific target locations in a FoI.

IV. Tessellation-based Connected *k*-Coverage

In this section, we investigate the problem of connected *k*-coverage in PWSNs using square tiles and deterministic sensing model. First, we tessellate a FoI using square tiles and generate a square tessellation. Second, we build specific region inside each of the square tile for sensor placement. Based on this sensor placement strategy, we compute the planar sensor density required to *k*-cover a planar FoI. Next, we compute the necessary relationship that must exist between the sensing radius r_s and communication radius r_c to ensure network connectivity. We apply this same approach to a stochastic sensing model.

A. Deterministic Sensing Model

Based on the generated square tessellation, we construct a cusp square area inside the square tile for sensor placement (Figure 1). and the strategy is to place *k* sensors in this cusp square area to achieve *k*-coverage of the square tile (Figure 2). Based on this strategy of sensor placement, we present Theorem 1, which states the sufficient condition for *k*-coverage using our square tessellation and its cusp square area configuration.

Theorem 1 (*k-Covered Field*) [20]: A FoI is *k*-covered if each of the square tiles of the tessellation has at least *k* active sensors placed in its corresponding cusp square area.

Lemma 1 below computes the *k*-coverage area for a square tile based on the cusp square area configuration.

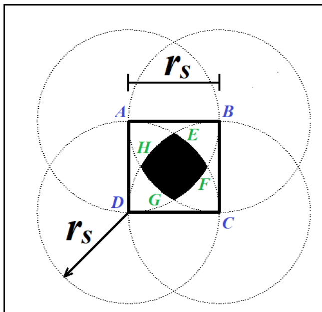

Fig. 1. Cusp square area of square tile.

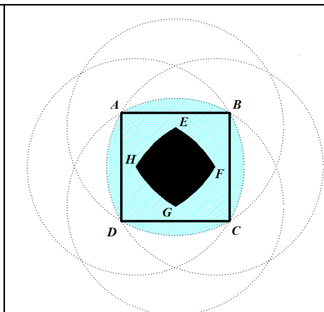

Fig. 2. *k*-coverage area of Square tile.

Lemma 1 (*k-Covered Area*) [21]: The *k*-covered area A_k formed by the intersection of the sensing disks of *k* sensors placed in a cusp square area of its corresponding square tile, can be computed as follows:

$$A_k = \left(\frac{2\pi + 3 - 3\sqrt{3}}{3}\right) r_s^2$$

where r_s is the radius of the sensing range of the sensors.

Theorem 2 (*Sensor Density*) [20]: The sensor density $\lambda(k, r_s)$, which is required to *k*-cover a FoI, is computed as follows:

$$\lambda(k, r_s) = \frac{0.734k}{r_s^2}$$

where r_s is the sensing radius of sensor, and $k \geq 1$.

Lemma 2 (*Network Connectivity*) [20]: A Square tessellation-based *k*-coverage configuration is said to be connected if the radii of sensing and communication ranges of the sensors, r_s and r_c, respectively, obey the inequality: $r_c \geq 2r_s$

B. Stochastic Sensing Model

Definition 1 (*Stochastic k-coverage*): A point *P* in FoI is said to be probabilistically *k*-covered if the probability of detection of an event occurring at *P* by at least *k* sensors is at least equal to certain threshold probability p_{th}, where $0 < p_{th} < 1$.

Theorem 3 (*Minimum k-coverage probability*): The minimum required probability of detection for *k*-coverage using our stochastic sensing model is given by

$$p_{min} = 1 - \left(1 - e^{-\beta r_s{}^\alpha}\right)^k$$

where r_s is the sensing radius of sensor.

Proof: In order to compute p_{min}, we identify the the least possibly *k*-covered point in a FoI. From Theorem 1, we deploy *k* sensors in the cusp square area to achieve *k*-coverage. We can clearly observe that point A is the least possibly *k*-covered if all the *k* sensors are deployed on the arc FG of the cusp square area. Also, point A is the farthest and equidistant point from the arc FG, and the distance is exactly r_s. Thus, the minimum required probability of detection p_{min} for the least possible *k*-covered point A, using our stochastic sensing model, is given by,

$$p_{min} = 1 - \prod_{i=1}^{k} (1 - p(P, s_i)) = 1 - \left(1 - e^{-\beta r_s{}^\alpha}\right)^k$$

In order to solve the stochastic *k*-coverage problem, we have to select a minimum subset S_{min} of sensors, where $S_{min} \subseteq S$, such that every point in a FoI is probabilistically *k*-covered by at least *k* sensors with probability of detection is at least equal to p_{th}. This allows us to compute the minimum stochastic sensing radius r_s^* that allows us to achieve stochastic *k*-coverage of a FoI with probability no less than p_{th}.

Lemma 3 (*Stochastic Sensing Radius*): The minimum stochastic sensing radius r_s^* that is required to achieve stochastic *k*-coverage of a FoI is computed as follows:

$$r_s{}^* = \left(-\frac{1}{\beta}\, ln\left(1 - (1 - p_{th})^{1/k}\right)\right)^{1/\alpha}$$

Proof: From Definition 1 and Theorem 3, we have,

$$p_{min} \leq p_{th}$$

$$\Rightarrow r_s \leq \left(-\frac{1}{\beta}\, ln\left(1 - (1 - p_{th})^{1/k}\right)\right)^{1/\alpha}$$

Thus, the minimum stochastic sensing radius r_s^* is given by:

$$r_s{}^* = \left(-\frac{1}{\beta}\, ln\left(1 - (1 - p_{th})^{1/k}\right)\right)^{1/\alpha}$$

Theorem 4 (*Stochastic Planar Sensor Density*): The stochastic planar sensor density $\lambda^*(k, r_s)$, which is required to *k*-cover a field of interest, is computed as follows:

$$\lambda^*(k, r_s) = \frac{0.734k}{(r_s^*)^2}$$

Lemma 4 (*Stochastic Network Connectivity*): A Square tessellation-based *k*-coverage configuration is said to be connected if the stochastic communication radius r_c^* and stochastic sensing radius r_c^*, obey the inequality: $r_c{}^* \geq 2r_s{}^*$

C. Stochastic k-Coverage Protocol

In the resulting stochastic *k*-coverage protocol, denoted by St-*k*-CSqu, we used the same sensor selection and scheduling strategies to minimize the sensors' energy consumption during the *k*-coverage process, thus, maximizing the network lifetime.

V. Performance Evaluation

In this section, we present the simulation results of our stochastic *k*-coverage protocol St-*k*-CSqu, using an open source high-level simulator [21] built using *C* and *Python* languages.

Fig. 3 shows the variation of stochastic planar sensor density λ^* (based on Theorem 4) with changing degree of coverage *k* for path loss exponent $\alpha = 2$ and $\alpha = 4$. We observe that λ^* increases with increase in *k* for constant α. Also, it is clear that for higher threshold probability p_{th} values, higher the λ^* to attain same level of *k* for constant α. For $\alpha = 2$, there is a slight deviation from expected behavior of λ^* versus *k*, while for $\alpha = 4$, the behavior of λ^* versus *k* is as expected, *i.e.,* λ^* is directly proportional to *k*. Fig. 4 shows the variation of degree of coverage *k* with regard to the number of active sensors n_a for our stochastic protocol for path loss exponent $\alpha = 2$ and $\alpha = 4$. We can clearly observe that a higher level of *k* is achieved using higher values of n_a. Similarly, number of active sensors n_a required for stochastic *k*-coverage process increases with increasing p_{th} and α values for achieving specific level of *k*. These experimental results shows a good match between theory and simulations. The plots in Fig. 5 consider different values of threshold probability p_{th} and path loss exponent α, with constant degree of coverage $k = 3$. They illustrate the behavior of the number of active sensors n_a required for stochastic *k*-coverage process with respect to the physical characteristics of sensor's sensing unit β (section II.B). From the plots, it is evident that n_a required increases with increase in β. Fig. 6 shown below compares the performance of our protocol St-*k*-CSqu with protocols SCP_k [5] and RCH_k [8]. It is clear that our protocol has higher operational network lifetime compared to other protocols for similar experimental conditions.

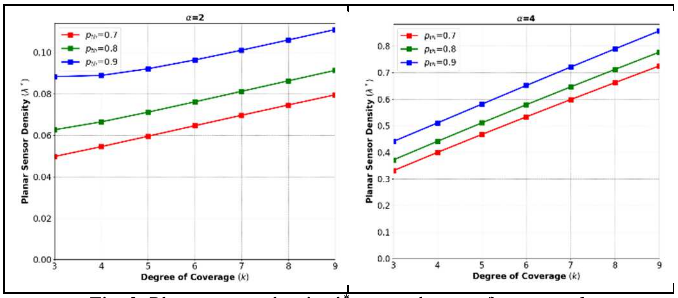

Fig. 3. Planar sensor density λ^* versus degree of coverage k

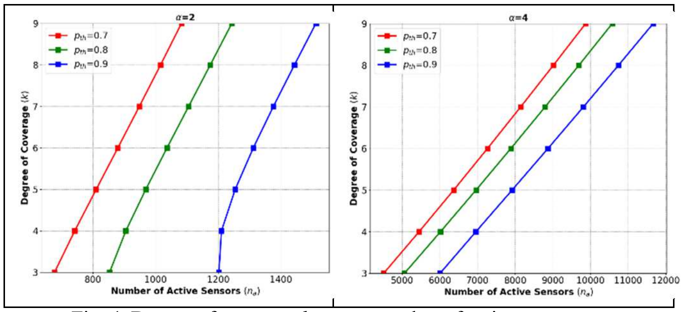

Fig. 4. Degree of coverage k versus number of active sensors n_a

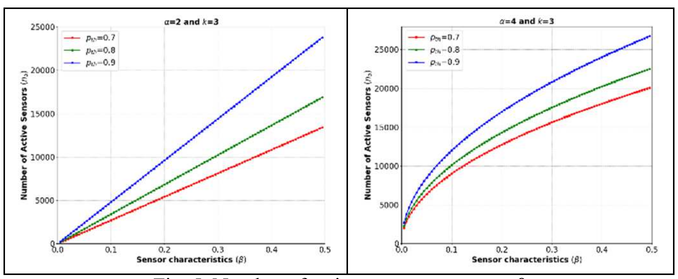

Fig. 5. Number of active sensors n_a versus β

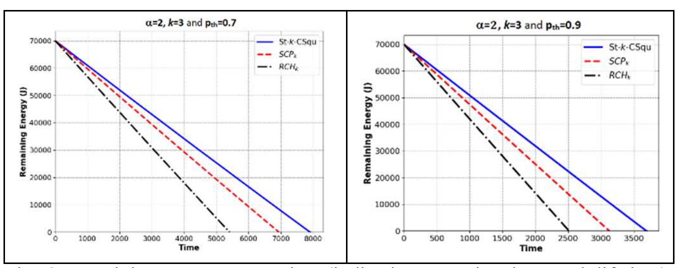

Fig. 6. Remaining energy versus time (indicating operational network lifetime).

VI. Conclusion

In this paper, we investigated the stochastic connected k-coverage problem in PWSNs using square tessellation-based approach. We proposed a stochastic protocol St-k-CSqu, which is an updated version of our k-CSqu protocol [20]. Our future work is three-fold. First, we plan to extend our square tessellation-based theory to heterogeneous sensors, where sensors may have different characteristics. Second, we want to extend our approach for solving stochastic connected k-coverage problem in three dimensional WSNs. Finally, we will be placing our protocols into practice using a real-world sensor-testbed.

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References

[1] N. Bulusu, J. Heidemann, and D. Estrin, “GPS-less low-cost outdoor localization for very small devices,” *IEEE Pers. Commun.*, 7(5), pp. 28–34, 2000.

[2] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, “An application-specific protocol architecture for wireless microsensor networks,” *IEEE Trans. Wirel. Commun.*, 1(4), pp. 660–670, 2002.

[3] F. Ye, G. Zhong, J. Cheng, S. Lu, and L. Zhang, “PEAS: a robust energy conserving protocol for long-lived sensor networks,” *Proc.* 23*rd International Conference on Distributed Computing Systems, 2003.*

[4] Y.-C. Wang and Y.-C. Tseng, “Distributed deployment schemes for mobile wireless sensor networks to ensure multilevel coverage,” *IEEE Trans. Parallel Distrib. Syst.*, 19(9), pp. 1280–1294, 2008.

[5] H. M. Ammari, “Stochastic *k*-coverage in wireless sensor networks,” in *Wireless Algorithms, Systems, and Applications*, Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 125–134.

[6] H. M. Ammari, “A unified framework for *k*-coverage and data collection in heterogeneous wireless sensor networks,” *J. Parallel Distrib. Comput.*, vol. 89, pp. 37–49, 2016.

[7] H. M. Ammari, “Achieving sensing *k*-coverage using hexagonal tiling: Are we done yet?,” in *2019 IEEE 16th International Conference on Mobile Ad Hoc and Sensor Systems (MASS)*, 2019.

[8] H. M. Ammari, “Connected *k*-coverage in two-dimensional wireless sensor networks using hexagonal slicing and area stretching,” *J. Parallel Distrib. Comput.*, vol. 153, pp. 89–109, 2021.

[9] J. Yu, S. Ren, S. Wan, D. Yu, and G. Wang, “A stochastic *k*-coverage scheduling algorithm in wireless sensor networks,” *Int. J. Distrib. Sens. Netw.*, 8(11), p. 746501, 2012.

[10] Z. Sun, C. Li, X. Xing, H. Wang, B. Yan, and X. Li, “K-degree coverage algorithm based on optimization nodes deployment in wireless sensor networks,” *Int. J. Distrib. Sens. Netw.*, 13(2), 2017.

[11] J. Yu, S. Wan, X. Cheng, and D. Yu, “Coverage contribution area based *k*-coverage for wireless sensor networks,” *IEEE Trans. Veh. Technol.*, 66(9), pp. 8510–8523, 2017.

[12] M. Krishnan, V. Rajagopal, and S. Rathinasamy, “Performance evaluation of sensor deployment using optimization techniques and scheduling approach for *K*-coverage in WSNs,” *Wirel. Netw.*, 24(3), pp. 683–693, 2018.

[13] M. Chenait, B. Zebbane, and N. Badache, “A new *k*-coverage model to determine redundant sensors in wireless sensor networks,” in *2018 International Conference on Smart Communications in Network Technologies (SaCoNeT)*, 2018.

[14] M. Elhoseny, A. Tharwat, X. Yuan, and A. E. Hassanien, “Optimizing *K*-coverage of mobile WSNs,” *Expert Syst. Appl.*, vol. 92, pp. 142–153, 2018.

[15] P. Hoyingcharoen and W. Teerapabkajorndet, “Expected probabilistic detection and sink connectivity in wireless sensor networks,” *IEEE Sens. J.*, 19(12), pp. 4480–4493, 2019.

[16] C. Naik and D. P. Shetty, “Differential evolution metaheuristic scheme for *k*-coverage and m-connected optimal node placement in wireless sensor networks,” *Int. J. Comput. Inf. Syst. Ind. Manag. Appl.,* vol. 11, pp. 132-141, 2019.

[17] F. Abbasi, A. Mesbahi, and J. Mohammadpour Velni, “A new Voronoi-based blanket coverage control method for moving sensor networks,” *IEEE Trans. Control Syst. Technol.*, 27(1), pp. 409–417, 2019.

[18] S. Harizan and P. Kuila, “Nature-inspired algorithms for *k*-coverage and m-connectivity problems in wireless sensor networks,” in *Lecture Notes in Networks and Systems*, Springer Singapore, 2020, pp. 281–301.

[19] P. Natarajan and L. Parthiban, “*k*-Coverage m-connected node placement using shuffled frog leaping: Nelder–Mead algorithm in WSN,” *J. Ambient Intell. Humaniz. Comput.*, 2020.

[20] K. Nakka and H. M. Ammari, “K-csqu: Ensuring connected K-coverage using cusp squares of square tessellation,” *SSRN Electron. J.*, 2022.

[21] *wsn: A Wireless Sensor Network simulator in Python and C++ (via SWIG)*, *Github.com*. [Online]. Available: https://github.com/darolt/wsn