Hierarchical Deployment and Square Tessellation for Connected k-Coverage in Heterogeneous Planar Wireless Sensor Networks

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Coverage and connectivity are the major performance metrics that reflect the quality of service provided by heterogeneous planar wireless sensor networks (PWSNs) monitoring a planar field of interest (PFoI), where the deployed sensors may not necessarily have the same capabilities in terms of their sensing range, communication range, and maximum battery-power capacity. Precisely, coverage is considered as the main functionality of PWSNs, which is meaningful only when connectivity is guaranteed. Therefore, it is important that both coverage and connectivity be jointly considered in the deployment of heterogeneous PWSNs. In order to account for joint coverage and connectivity, we propose to solve the problem of connected k-coverage in heterogeneous PWSNs, where every point in a PFoI is covered by at least k sensors simultaneously, while all the deployed sensors are mutually connected, either directly or indirectly, with $k > 1$. While most existing studies of this problem focus on homogeneous sensors, which have the same above-mentioned capabilities (i.e., initial energy, sensing range, and communication range), our study in this article considers heterogeneous ones. More specifically, we propose a hierarchical (or multitier) deployment of heterogeneous sensors in a square FoI, which is divided into concentric square bands with the same width difference to achieve k-coverage of this PFoI. Based on this multitier sensor deployment and the slicing of a square FoI into square bands for k-coverage, we establish the necessary relationship for connectivity among the sensors located in adjacent bands. Finally, we propose our heterogeneous k-coverage protocol and validate our theoretical analysis using simulation results. We find that the deployment of heterogeneous sensors helps achieve much better results compared to those obtained using homogeneous sensors. Furthermore, our proposed protocol outperforms an existing connected k-coverage protocol for heterogeneous PWSNs with respect to various performance metrics.

CCS Concepts: • **Networks → Network algorithms**; **Network protocol design**; **Network performance evaluation**; **Ad hoc networks**;

Additional Key Words and Phrases: Planar wireless sensor networks, k-coverage, connectivity, sensor heterogeneity, hierarchical deployment, square tessellation

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1 Introduction

A **planar wireless sensor network (PWSN)** is an infrastructureless wirelessly connected network, which consists of tiny and low-powered sensors, dispersed in a **planar field of interest (PFoI)**. These sensors communicate with each other in a multihop fashion and relay their sensed data to a central entity, called the *sink*, for further analysis and processing. The sensing capabilities of sensors made them capable of detecting and measuring various environmental parameters, such as sound, light, temperature, pressure, pollutants, and vibrations, to name a few. Also, these sensors are deployable for a wide range of applications, including health and environment monitoring, seismic monitoring, industrial process automation, and military surveillance. Moreover, owing to their communication capabilities, these sensors are able to relay the collected/sensed data to the sink. Therefore, for the successful monitoring of PFoI and data collection as well as data transmission to the sink, these PWSNs should ensure both coverage and connectivity during their operational lifetime.

It is clear that PWSNs should be designed in such a way that there are no coverage and connectivity holes, while they are monitoring a PFoI. The major problem in the design of PWSNs is the scarce resources of the sensors, such as limited battery power (or energy), CPU, storage, and bandwidth, to name a few, with energy being the most critical aspect. Moreover, some critical applications, such as intruder detection and tracking, forest fire monitoring, flood monitoring, and precision agriculture, require that every point of a PFoI be covered (or sensed) by more than one sensor simultaneously. This type of redundant coverage ensures fault-tolerant data collection during the network operation. Thus, in this article, we focus on a more general concept of coverage, called k-coverage. Precisely, considering both coverage and connectivity, we attempt to solve the problem of connected k-coverage in PWSNs using heterogeneous sensors for providing a more realistic and accurate view on the design of PWSNs. Next, we discuss our motivations to work on the connected k-coverage problem in heterogeneous PWSNs (Section 1.1). Also, we state this problem in detail (Section 1.2). Then, we summarize our contributions in this article (Section 1.3).

1.1 Motivations

Our motivations to investigate the problem of connected k-coverage in PWSNs, while considering heterogeneous sensor deployment are based on the following observations. First, the k-coverage problem in PWSNs has been extensively studied in the literature [8–26], most of which using homogeneous sensors. This imposes a severe restriction on the design and development of real-world WSNs as all the sensors are required to have the same characteristics listed above, which is unrealistic. However, in real-world sensing applications, PWSNs have heterogeneous sensors, which have varied features (i.e., sensing range, communication range, energy, storage, computation, etc.). These heterogeneous sensors-based PWSNs have the potential to enhance the operational network lifetime and reliability without significant increase in cost. Second, in the design of WSNs, such as triangulation-based positioning systems [5], multisensor data fusion [6], and space exploration [7], a degree of coverage $k \geq 3$ is required.

1.2 Problem Formulation

In this article, we want to investigate the problem of connected k-coverage in heterogeneous PWSNs, where each point in a PFoI is simultaneously sensed (or covered) by at least k

heterogeneous sensors, while all the sensors participating in the k-coverage are mutually connected, such that any pair of sensors can freely communicate with each other, where $k > 1$ is called the *degree of coverage*. Precisely, we intend to solve this problem by addressing the following interrelated questions:

— **Question 1:** What is the deployment strategy of heterogeneous sensors in a PFoI, such that every point in the PFoI is sensed (or covered) by at least k sensors simultaneously, using a minimum number of sensors?
— **Question 2:** What is the corresponding heterogeneous planar sensor density (i.e., number of heterogeneous sensors per unit area) to achieve k-coverage of a PFoI?
— **Question 3:** What is the necessary relationship between the sensing radius $r_{s,x}$ and the communication radius $r_{c,x}$ of a heterogeneous sensor s_x for ensuring network connectivity?
— **Question 4:** What is the sensor scheduling strategy for heterogeneous sensors to k-cover a PFoI, while minimizing the average total energy consumption by the active sensors?

It is worth noting that the problem of connected k-coverage in PWSNs using homogeneous sensors is NP-hard [24]. With heterogeneous sensors, this problem is also NP-hard. Intuitively, the NP-hardness of this problem with homogeneous sensors leads to its NP-hardness with heterogeneous sensors.

1.3 Contributions

In our study, we consider the concept of square tessellation and deploy heterogeneous sensors hierarchically in a PFoI with the goal of attaining connected k-coverage in PWSNs using a minimum set of active heterogeneous sensors. The major contributions of this article can be summarized as follows:

— First, we propose a hierarchical deployment of heterogeneous sensors, where a square PFoI is decomposed into n concentric square bands. These concentric square bands have increasing widths and the width difference w_d between any two adjacent square bands is constant. The innermost square band has a width of ${r_s}^{\min}$ and the outermost square band has a width of ${r_s}^{\max}$, where ${r_s}^{\min}$ and ${r_s}^{\max}$ stand for the minimum and maximum sensing radii, respectively, of the heterogeneous sensors. Moreover, the values of n and w_d are dependent on the side length L of the square PFoI and minimum and maximum sensing radii, ${r_s}^{\min}$ and ${r_s}^{\max}$, respectively, of the deployed heterogeneous sensors. Then, we deploy homogeneous sensors of sensing radius $r_{s,x}$ in the square band b_x, such that any two concentric square bands have different sets of sensors leveraging the concept of sensor heterogeneity. This type of sensor deployment helps achieve heterogeneous connected k-coverage of a PFoI through ensuring homogeneous connected k-coverage of each square band.
— Second, we tessellate each square band b_x using adjacent and nonintersecting square tiles of side length equal to the sensing radius $r_{s,x}$ of the sensors deployed in that specific square band b_x. Then, we construct a cusp-square area inside every square tile, in order to ensure k-coverage of each square tile, thus collectively attaining k-coverage of that square band b_x, where $k > 1$.
— Third, we compute the planar sensor density corresponding to our hierarchical deployment of heterogeneous sensors, leveraging concentric square bands and square tessellation, to k-cover the entire PFoI using heterogeneous sensors, where $k > 1$.
— Fourth, based on our proposed hierarchical heterogeneous sensor deployment, we establish the relationship that should exist between the sensing radius $r_{s,x}$ and the communication radius $r_{c,x}$ of the sensors deployed in a square band b_x, such that all the active sensors participating in the k-coverage process are mutually connected across all concentric square

bands, thus, achieving connected k-coverage in PWSNs using heterogeneous sensors, where $k > 1$.

— Fifth, we propose a centralized heterogeneous sensor selection protocol, where the selected sensors are duty cycled (or scheduled) to ensure connected k-coverage of a PFoI, leveraging our hierarchical deployment of heterogeneous sensors and square tessellation.
— Sixth, we evaluate the performance of our proposed connected k-coverage protocol using simulations in comparison to our theoretical analysis. We find a very close-to-perfect match between our theoretical results and the obtained simulation ones.

1.4 Organization

The remainder of this article is structured as follows. In Section 2, we discuss related work on the problem of coverage in homogeneous and heterogeneous PWSNs along with their limitations. In Section 3, we introduce the fundamental concepts and define our network and energy models, which are used in our analysis of the connected k-coverage problem in heterogeneous PWSNs. In Section 4, we discuss our homogeneous sensors-based solution to the k-coverage in PWSNs [8] and its possible extension to k-cover a PFoI along with its limitations. Also, we propose a hierarchical deployment approach of heterogeneous sensors so as to overcome the previously discussed limitations and solve the connected k-coverage problem in heterogeneous PWSNs. Leveraging this hierarchical sensor deployment approach, we compute the corresponding planar sensor density required for k-coverage of a PFoI using heterogeneous sensors. In addition, we determine the necessary relationship between the sensing and communication ranges of the sensors for network connectivity in order to achieve connected k-coverage of a PFoI using heterogeneous sensors. In Section 5, we discuss our proposed centralized heterogeneous connected k-coverage protocol, called Het-k-CSqu. In Section 6, we present the simulation results of our protocol Het-k-CSqu and compare these results with our theoretical results as well as those obtained using homogeneous sensors. Furthermore, we assess the performance of our protocol Het-k-CSqu in comparison with an existing centralized heterogeneous protocol, called PR-Het-CCC$_k$ [31]. In Section 7, we conclude and present possible extensions of our proposed work in this article.

2 Related Work

In this section, we briefly discuss various approaches addressing the coverage and k-coverage problems in PWSNs. First, we present some solutions to these problems using homogeneous sensors (Section 2.1). Second, we review a sample of approaches solving these problems using heterogeneous sensors (Section 2.2). Third, we show the possible limitations of all these existing approaches in comparison to our proposed approach (Section 2.3).

2.1 Homogeneous Sensor Deployment

Yu et al. [12] studied the properties of regular pentagons and developed a stochastic k-coverage theory, which models the sensing range of the sensors using four regular pentagons with central areas, where k-coverage of a sensor's sensing range is achieved by placing $k - 1$ sensors collectively in those four central areas. Wang et al. [9, 10] have developed a k-coverage eligibility algorithm for sensor state scheduling by evaluating the coverage of the intersection points of sensing range of a sensor with that of its neighbors. They have proposed a **coverage configuration protocol (CCP)** by extending their k-coverage eligibility algorithm. Qiu et al. [13] have developed a k-order **local k-coverage Voronoi diagram (LVD)**, which checks the precision of the critical points of a sensor and allocates neighboring sensors to an under k-covered sensor for achieving the expected degree of coverage k. They proposed a **distributed Voronoi-based cooperation (DVOC)** scheme that ensures k-coverage using k-order LVDs and leverages k-order Delaunay triangles for mitigating the coverage voids that may arise. Chenait et al. [16] have developed the sector redundancy determination algorithm that slices the sensing range of sensors into sectors based on a predefined angle and evaluates the sensors for the k-coverage process. Leveraging this algorithm, they have proposed SRA-Per and SRA-SP protocols.

Ammari and Das [11] studied the geometrical properties of the Reuleaux triangle and developed a theory to address the connected k-coverage problem in PWSNs, which leverages the Reuleaux triangle-based tessellation. This theory ensures k-coverage of two adjacent Reuleaux triangles by placing k sensors in their overlapping area, called lens. Based on this theory, Ammari and Das [11] proposed deterministic randomized k-coverage protocols CERACC_k and DIRACC_k; and deterministic clustering-based k-coverage protocols T-CRACC_k and D-CRACC_k. Also, Ammari [31] has developed a stochastic k-coverage protocol SCP_k [31] for PWSNs. Krishnan et al. [19] utilized four optimization schemes, namely, heuristic algorithm, **artificial bee colony (ABC)** algorithm, **ant colony optimization (ACO)** algorithm, and **particle swarm optimization (PSO)** algorithm, for sensor placement and leveraged minimum dominating set-based heuristics for sensor duty cycling. They proposed 10 different protocols by generating various combinations of these sensor placement and sensor duty-cycling schemes and evaluated their performances. Sun et al. [15] have leveraged the process of node deployment optimization for addressing the k-coverage problem. Qin and Chen [18] have proposed a binary **differential evolution (DE)** –based area coverage algorithm that achieves a specific coverage demand by searching an improved subset of sensors in the network.

Elhoseny et al. [20] leveraged a **genetic algorithm (GA)** for attaining k-coverage of target locations in a PFoI, with an objective of maximizing the operational network lifetime. Leveraging Ammari's Reuleaux triangle-based tessellation, Yu et al. [14] constructed a **coverage contribution area (CCA)** for sensor deployment and proposed various k-coverage protocols, namely, SCRT-PCA_k, DCRT-PCA_k, and DIRT-PCA_k. Torshizi and Sheikhzadeh [26] developed a distributed algorithm, called CLARRKC, which duty cycles the sensors using **cellular learning automata (CLA)** to remove redundant active sensors and ensure the use of an optimal number of sensors during the k-coverage process. Niak and Shetty [21] also utilized the DE algorithm for computing the optimal candidate locations for sensor deployment in a PFoI with an objective of achieving k-coverage of target locations in a PFoI. Nakka and Ammari [39] developed the k-InDi protocol for achieving connected k-coverage in PWSNs using hexagonal tessellation. Taking heuristics and nature-inspired algorithms into consideration, Harizan and Kuila [22] leveraged GA, PSO, DE, and gravitational search algorithms for determining the optimal sensor placement locations such that k-coverage of target locations in a PFoI is achieved. Elloumi et al. [25] proposed two mixed linear programming-based solutions for the problem of target k-coverage, leveraging the Single commodity flow model and the Miller-Tucker-Zemlin model. Natarajan and Parthiban [23] addressed the target k-coverage problem in PWSNs by determining the optimal node positions using the shuffled frog leaping Nelder-Mead algorithm. Alibeiki et al. [24] also leveraged the GA approach for addressing the problem of k-coverage in directional PWSNs of both overprovisioned and underprovisioned sensor configurations.

2.2 Heterogeneous Sensor Deployment

Tarnaris et al. [27] have addressed both target coverage and target k-coverage problems in heterogeneous PWSNs by leveraging GA and PSO algorithms for determining the optimal sensor deployment locations in a PFoI with an objective of achieving a specific degree of coverage. Ammari [31] has proposed a pseudo-random (or multitier) deployment of heterogeneous sensors for a circular PFoI and leveraged his Reuleaux triangle-based k-coverage theory for achieving connected k-coverage of the circular PFoI. Based on the combination of pseudo-random deployment and

Reuleaux triangle-based k-coverage, Ammari [31] has proposed deterministic heterogeneous protocols, called PR-Het-CCC$_k$ and PR-Het-DCC$_k$. Li et al. [35] proposed a two-stage coverage control algorithm for heterogeneous directional PWSNs using a stepwise method for adjusting the working directions of sensors, and leveraging a virtual force-directed self-adaptive DE algorithm for sensor deployment in a PFoI, such that k-coverage of the targets is achieved. Zishan et al. [33] have addressed the target coverage problem in heterogeneous visual PWSNs by formulating the problem using both **integer linear programming (ILP)** and **integer quadratic programming (IQP)** formulations. They observed that the IQP-based solution outperformed the ILP-based solution to the target coverage problem in heterogeneous visual PWSNs. In addition, in order to address this problem in large networks, they proposed a greedy quadratic heuristics combined IQP solution.

Charr et al. [29] have studied the problem of energy management under target coverage constraints in heterogeneous PWSNs and proposed two solutions by addressing the **heterogeneous disjoint set covers (HDSCs)** –based scheduling using a mixed linear integer programming formulation and GA-based approach. Liu and Ouyang [32] studied the problem of k-coverage in heterogeneous camera PWSNs where the sensors are deployed outside the boundaries of the PFoI. They developed a computational geometry-based k-coverage theory that estimates the minimum number of camera sensors required to achieve the k-coverage of the PFoI. Manju et al. [38] proposed an energy-based greedy heuristic scheme called maximum coverage small lifetime that restricts the usage of sensors that poorly covers the targets and promotes the usage of sensors that have a maximum residual energy and coverage rate, to achieve target k-coverage in heterogeneous PWSNs. Cao et al. [36] proposed an improved optimization algorithm that combines a social spider optimization scheme and a chaotic optimization scheme in order to improve coverage and minimize the redundant coverage for heterogeneous PWSNs.

Mcheick et al. [34] proposed a **coverage maximization of heterogeneous wireless networks (CMHWNs)** algorithm, which leverages the intersection points of the sensing ranges of the heterogeneous sensors in order to maximize the coverage with a constraint of achieving m-connected sensors. Li et al. [28] designed a bidirectional mutation-based hybrid GA algorithm for extending the operational network lifetime of heterogeneous PWSNs with the objective of maintaining the expected coverage ratio. Rahmani et al. [30] developed a multistage area coverage algorithm that leverages a fuzzy scheduling mechanism for sensor duty cycling and a shuffled frog leaping algorithm for the best placement of mobile sensors, such that the coverage rate of voids is maximized. Sangaiah et al. [37] proposed a bat algorithm–based solution for the selection of the optimal number of sensors to cover specific targets in a PFoI as well as increasing the network lifetime using heterogeneous sensors.

2.3 Discussion

As stated earlier, most of the research related to coverage and k-coverage uses homogeneous sensors [8–26]. Even the latest research, leveraging computational intelligence algorithms [18–26], compute optimal solutions for these coverage and k-coverage problems using the homogeneous sensor constraint. This places an unrealistic restriction on the design of real-world WSNs, most of which consist of heterogeneous sensors that have varied characteristics, and, particularly, their initial battery power, sensing range, and communication range. Furthermore, many existing works in homogeneous PWSNs are unable to achieve full connectivity of all active sensors and their solutions do not address coverage void issues.

There are researchers who leveraged computational intelligence algorithms [27–30, 35–37] for addressing the problems of coverage and k-coverage of specific targets in a PFoI using heterogeneous sensors. In a recent work by Rahmani et al. [30], the proposed algorithm requires a separate optimization algorithm for mitigating the coverage voids in a PFoI. Similar to most homogeneous sensors-based solutions, many of the existing counterparts using heterogeneous sensors are also unable to ensure full connectivity of all active sensors. However, our hierarchical sensor deployment combined with PFoI tessellation ensures connected k-coverage, where a PFoI is k-covered while all active heterogeneous sensors are mutually connected. Moreover, achieving connected k-coverage by our approach using heterogeneous sensors ensures reliable monitoring of a PFoI, fault-tolerant data collection by sensors without any coverage voids, and reliable connectivity of all active sensors without any connectivity voids.

It is worth noting that the work by Ammari [31] was able to develop a reliable connected k-coverage solution for heterogeneous PWSNs. However, in comparison with our hierarchical deployment of heterogeneous sensors, the pseudo-random deployment [31] developed by Ammari fails to achieve optimality. Indeed, Ammari's deployment strategy [31] requires that the innermost band have a larger width and subsequent bands have decreasing widths. Considering Ammari's Reuleaux triangle tessellation-based k-coverage approach [31], we can easily infer that a fewer number of larger Reuleaux triangle tiles tessellate smaller-area bands and a higher number of smaller Reuleaux triangle tiles tessellate larger-area bands. Thus, increasing the overall total number of tiles of a PFoI implicitly increases the total number of active sensors required for any tessellation-based k-coverage of a PFoI. But our hierarchical sensor deployment strategy ensures that the innermost band has a smaller width and the subsequent bands have increasing widths. Furthermore, our square tessellation-based k-coverage approach ensures that a fewer number of smaller square tiles tessellate smaller-area bands and a higher number of larger square tiles tessellate larger-area bands, thus decreasing the overall total number of tiles of a PFoI as well as the total number of active sensors required for k-coverage. Our simulation results in Section 6 prove these inferences.

3 Preliminaries and Models

In this section, we introduce the specific terminology that is used throughout this article. Also, we describe our energy model and network model, along with their assumptions, which are used in our analysis of the connected k-coverage problem in heterogeneous PWSNs.

3.1 Terminology

Definition 1 (Sensing Range). The sensing range of a sensor s_i is the area A_s around it, such that the sensor s_i can detect any event occurring in the area A_s.

Definition 2 (Communication Range). The communication range of a sensor s_i is the area A_c around it, such that the sensor s_i can communicate with any other sensor present in the area A_c.

Definition 3 (k-coverage). A PFoI is k-covered if every point in it is sensed (or covered) by at least k sensors simultaneously. A PWSN monitoring such a k-covered PFoI is said to provide k-coverage, where k is the degree of coverage achieved by the PWSN with $k > 1$.

Definition 4 (Connected k-coverage). A PWSN monitoring a PFoI is said to provide connected k-coverage if and only if this PFoI is k-covered while all the sensors involved in the k-coverage process are connected to each other directly or indirectly.

Definition 5 (Planar Sensor Density). The planar sensor density is the number of heterogeneous sensors per unit area required to k-cover a PFoI by a PWSN.

3.2 Network Model

Assumption 1 (Location Awareness). *All the heterogeneous sensors deployed in a PFoI are aware of their locations through a* ***global positioning system (GPS)*** *or any localization technique [1].*

Assumption 2 (Sensor Deployment). *All the heterogeneous sensors are deployed uniformly and randomly in a square PFoI, unless otherwise specified.*

Assumption 3 (Sensor Heterogeneity). *All the deployed sensors in a PFoI have varied characteristics, such as their sensing range, communication range, and initial battery power (or energy).*

Assumption 4 (Deterministic Model). *The sensing and communication ranges of a sensor s_i are modeled by disks of radii $r_{s,i}$ and $r_{c,i}$, respectively, where the centers of both ranges coincide with the location of the sensor s_i.*

Assumption 5 (Sensor Mobility). *All the heterogeneous sensors are mobile and have the capability to freely move to specific locations in a PFoI.*

3.3 Energy Model

We have used the energy model proposed by Heinzelman et al. [2] for calculating the energy consumed by the sensors, while performing the activities of data transmission and data reception, as follows:

$$E_t\left(d\right) = \kappa\left(\varepsilon d^{\alpha} + E_e\right), \tag{1}$$

$$E_r = \kappa E_e, \tag{2}$$

where $E_t(d)$ is the energy consumed by a sensor s_i for transmitting a message of κ bits over a distance d, E_r is the energy spent by a sensor s_i for receiving a message of size κ bits, E_e is the electronic energy, $\varepsilon \in \{\varepsilon_{fs}, \varepsilon_{mp}\}$ is the transmitter amplifier in the free space (ε_{fs}) or multipath (ε_{mp}) model, and $\alpha \in [2,4]$ is the path-loss exponent.

The energy consumed by the sensors for performing the sensing activities is estimated based on the energy model proposed by Ye et al. [3]. In an ideal case, a sensor utilizes 0.012 J of energy (E_{idle}) in idle mode, 0.0003 J of energy (E_{sleep}) in sleep mode, and the energy consumed per distance moved for sensor movement (E_{move}) is randomly picked from the interval [0.008, 0.012] J/m [4], such that the total energy consumption for sensor mobility E_m is computed as

$$E_m = d_{\text{move}} E_{\text{move}}. \tag{3}$$

4 Heterogeneous *k*-Coverage

In this section, we investigate the problem of connected k-coverage in heterogeneous PWSNs, where the sensors have varied characteristics in terms of their sensing range, communication range, and initial energy. Next, in Section 4.1, we discuss our previous results on connected k-coverage in homogeneous PWSNs [8] using a specific configuration of square tessellation. In Section 4.2, we discuss the possible ways of extending our homogeneous sensors-based results and their limitations. Finally in Section 4.3, we present the best sensor deployment strategy for heterogeneous sensors for attaining connected k-coverage, while leveraging our previously proposed square tessellation [8].

4.1 Our Previous Research Results

For this article to be complete and self-contained, we briefly discuss our previous results with regard to the connected k-coverage problem in homogeneous PWSNs, using square tessellation, without their proofs. For more detailed explanations of these results, the interested reader is referred to our earlier work published in [8]. First, we tessellate a PFoI using square tiles of side length s_t whose value is equal to the radius r_s of the sensors' sensing range as shown in Figure 1. Then, we construct the sensor placement area for a square tile as follows: We draw four (4) circles

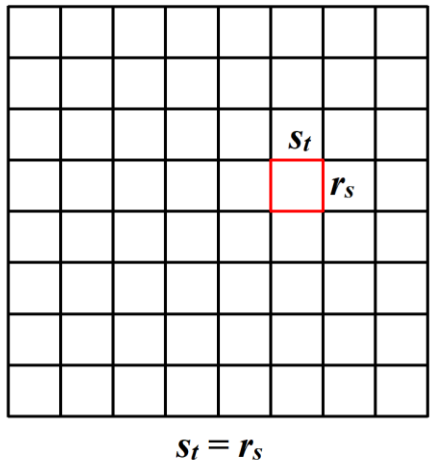
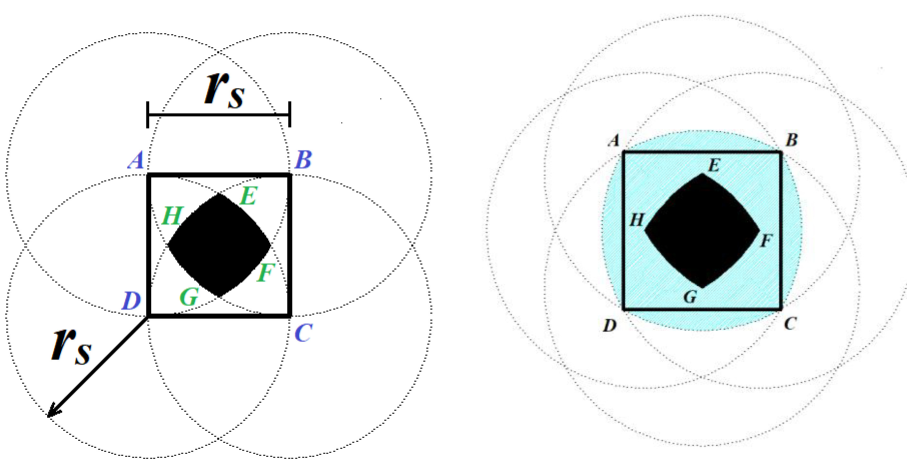


Fig. 1. Square tessellation of side length r_s..



(a) (b)

Fig. 2. (a) Cusp-square configuration for a square tile and (b) *k*-coverage area for a square tile.

of radius r_s centered at each vertex of the square tile as shown in Figure 2(a). The intersection area of these four circles forms the sensor placement area for our square tile, which is called the *cusp-square area*. Lemma 1 below gives the upper bound on the distance between any vertex (A/B/C/D) of square tile and any vertex (E/F/G/H) of cusp-square area as shown in Figure 2(a).

Lemma 1 (Largest Distance [8]). *For a square tile T of side length r_s, the maximum Euclidean distance between two points X and Y is r_s, where the domain of X is set of vertices of T and the domain of Y is set of vertices of cusp-square area of T. That is, $\delta(X, Y) \leq r_s$, where $X \in$ {vertices of square tile T}, $Y \in$ {vertices of cusp-square area of square tile T}, δ is the Euclidean distance function, and r_s is the radius of the sensors' sensing range.*

Leveraging the results of Lemma 1, Lemma 2 below establishes the necessary condition for *k*-coverage of a square tile of our tessellation.

Lemma 2 (Square Tile k-coverage [8]). *A square tile T of side length r_s is k-covered if and only if there are k active sensors of sensing radius r_s present in its corresponding cusp-square area, where r_s is the radius of the common sensing range of all the deployed homogeneous sensors.*

Exploiting Lemma 2, Theorem 1 below provides a sufficient condition for k-coverage of a PFoI. This is visually illustrated in Figure 2(b).

Theorem 1 (κ-covered Field [8]). *A PFoI is k-covered if each and every square tile of the tessellation has at least k active sensors present in the corresponding cusp-square area of each tile.*

Next, we discuss the possible ways of extending our previous results for k-coverage in homogeneous PWSNs [8] in order to investigate the problem of k-coverage in heterogeneous PWSNs. Moreover, for a better handling of the connected k-coverage problem in heterogeneous PWSNs, we discuss all the issues associated with the proposed extension of these results.

4.2 Extension to Heterogeneous Sensors

In general, sensors constituting a PWSN may not be homogeneous, i.e., they may have varied characteristics in terms of their communication range, sensing range, and initial battery power. For a precise solution to the problem of connected k-coverage in PWSNs, it is essential to account for both k-coverage and connectivity of all active sensors. Based on our previous research [8], the latter (i.e., connectivity) depends on the relationship that should exist between the sensing and communication radii of the sensors, whereas the former (i.e., k-coverage) depends solely on the sensing radius of the sensors. Therefore, we need to focus on the sensing radius of the sensors for solving the connected k-coverage problem in PWSNs. Considering sensor heterogeneity, we distinguish two special cases: One with the smallest sensing radius and the other one with the largest sensing radius among all the heterogeneous sensors. In the subsequent sections, we discuss each of these two particular cases.

4.2.1 Case 1: Tile length = smallest sensing radius ${r_s}^{min}$. Let ${r_s}^{\min}$ be the smallest sensing radius amongst all the heterogeneous sensors. In this scenario, the sink tessellates a PFoI using adjacent and nonintersecting square tiles of side length ${r_s}^{\min}$ and selects k sensors per tile for ensuring k-coverage of the PFoI. This could lead to an *over k-coverage* problem, where the sensors with sensing radius larger than ${r_s}^{\min}$ have the ability to cover some areas that are already k-covered.

Problem 1 (Over k-coverage). *Some other already k-covered areas $\{A_i, i = 1 \ldots n\}$ in a PFoI will also be covered by some powerful sensors whose sensing radius is larger than the smallest sensing radius ${r_s}^{min}$. This will happen although these powerful sensors are not selected by the sink to k-cover those already k-covered areas $\{A_i, i = 1 \ldots n\}$ in the PFoI. In other words, each of these powerful sensors is, in reality, selected to k-cover a certain area B_j, $\{B_j, j = 1 \ldots m\}$, which is not part of the set $\{A_i, i = 1 \ldots n\}$. However, because they have large sensing radii, these powerful sensors are able to cover additional areas.*

This over k-coverage problem may negatively impact the performance of PWSNs if there are fewer sensors with sensing radius ${r_s}^{\min}$ compared to the total number of heterogeneous sensors deployed to k-cover a PFoI. That is, the sensor heterogeneity, which may be leveraged to improve the performance of PWSNs, becomes an issue rather than a solution. In particular, a square tessellation with side length ${r_s}^{\min}$ would consist of a large number of tiles, which in turn increases the total number of active sensors required to k-cover a PFoI. This total number of active sensors would be the same regardless of whether they are the most powerful (i.e., sensing range $r_s = {r_s}^{\max}$), the least powerful sensors (i.e., $r_s = {r_s}^{\min}$), or in between (i.e., sensing range r_s satisfying ${r_s}^{\min} < r_s < {r_s}^{\max}$). However, as the square tessellation's tile side length is ${r_s}^{\min}$, a PFoI is guaranteed to be k-covered and all the active sensors are surely connected with each other during the overall network operation, based on our previous results [8].

4.2.2 *Case 2: Tile Length = Largest Sensing Radius r_s^{max}.*

Let $r_s^{\max}$ be the largest sensing radius amongst all the heterogeneous sensors. In this scenario, the sink tessellates the PFoI using adjacent and nonintersecting square tiles of side length $r_s^{\max}$ and selects k sensors per tile for ensuring k-coverage of a PFoI. This sensor selection process could lead to an *under k-coverage* problem, where the selected sensors with sensing radius smaller than $r_s^{\max}$ are unable to cover some areas in their assigned tiles in a PFoI. Also, we may have a *skewed communication* problem, where some sensors with a sensing radius smaller than $r_s^{\max}$ may not be capable of communicating with those most powerful sensors whose sensing radius is $r_s^{\max}$. Indeed, in general, a small sensing range implies a small communication range and that is why this skewed communication problem *may* occur.

Problem 2 (Under k-coverage). *In a square tessellation of side length r_s^{max}, if the sensors selected by the sink to participate in the k-coverage of a PFoI possess a sensing radius that is smaller than r_s^{max}, then they will not be able to cover some areas for which they are selected to k-cover.*

From Lemma 2 above, it is clear that a square tile of side length $r_s^{\max}$ will be k-covered if and only if there are k active sensors of sensing radius $r_s^{\max}$ in its corresponding cusp-square area. This implies that if there is at least one sensor with sensing radius lesser than $r_s^{\max}$ in a tile, then that tile surely cannot be k-covered. Thus, from Theorem 1, such a PFoI with square tessellation of side length $r_s^{\max}$ cannot be completely k-covered and thus arises the issue of under k-coverage of the underlying PFoI. To address this issue, more active sensors would be required to k-cover these under k-covered tiles. Therefore, the total number of active sensors would increase heavily if more sensors with a sensing radius smaller than $r_s^{\max}$ are selected for the current k-coverage round by the sink, thus increasing the energy consumption of active sensors per k-coverage round and decreasing the operational network lifetime. Moreover, it is clear that the total number of active sensors required to k-cover a PFoI may not be the same for different k-coverage rounds during the operation of a PWSN.

Problem 3 (Skewed Communication). *In a square tessellation of side length r_s^{max}, some sensors with sensing radius smaller than r_s^{max}, which are selected by the sink for a k-coverage round, may not be able to communicate with other active sensors located in neighboring tiles.*

Apart from sensing coverage, network connectivity is also an essential aspect that is needed for the correct operation of a PWSN. Considering a square tessellation of side length $r_s^{\max}$, which specifically requires active sensors of sensing radius $r_s^{\max}$, some of the sensors that are selected in certain tiles for a k-coverage round may not be connected to each other. For instance, consider two sensors, such that sensor s_x has a sensing radius $r_{s,x}$ and a communication radius $r_{c,x}$, and sensor s_y has a sensing radius $r_{s,y}$ and a communication radius $r_{c,y}$, where $r_{s,x} > r_{s,y}$ and $r_{c,x} > r_{c,y}$. Also, consider a scenario where sensor s_y lies in the communication range of the sensor s_x while sensor s_x is outside the communication range of sensor s_y. This scenario creates a skewed communication between the sensors s_x and s_y. In fact, sensor s_x will be able to send its collected data to the sink through its one-hop neighbor sensor s_y, but sensor s_y will not be able to do so with sensor s_x. Thus, due to these two problems, a square tessellation with side length $r_s^{\max}$, will not be able to achieve connected k-coverage for a heterogeneous sensor configuration. Hence, this draws more attention to Question 1 stated in Section 1.2 of how the sensors should be deployed in a PFoI, while leveraging sensor heterogeneity in order to improve the overall performance of PWSNs.

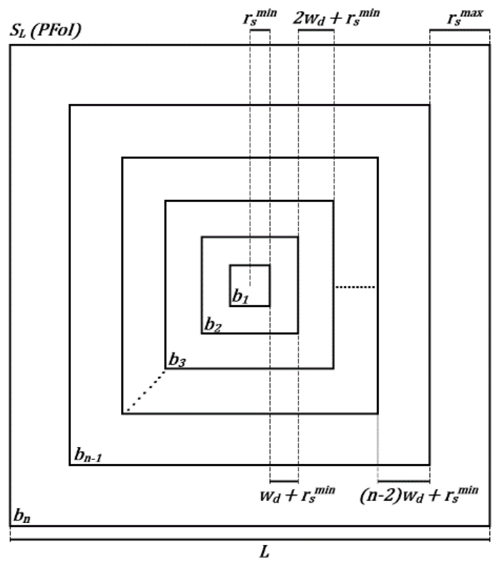


Fig. 3. PFoI decomposed into concentric square bands.

4.3 Hierarchical Sensor Deployment

We consider a square deployment field and heterogeneous sensors, which have varied characteristics in terms of their sensing range, communication range, and initial battery power. We propose to deploy these heterogeneous sensors in a hierarchy (or multitier) such that the most powerful sensors are near the boundary of a PFoI. Let us assume that the minimum and maximum sensing radii of the deployed sensors are $r_s^{\min}$ and $r_s^{\max}$, respectively, and the side length of the square deployment field is L. We propose to decompose this square field, denoted by S_L, into n concentric square bands of strictly increasing widths starting from the innermost band. Each of these bands has homogeneous sensors whose sensing radius is equal to the band's width, and any two different bands have two different sets of sensors. Precisely, the closest band to the center of the square field has the least powerful sensors compared to all other bands, and the farthest band has the most powerful sensors. Also, we assume that the difference between the widths of any two adjacent bands is constant and is denoted by w_d. More specifically, the width of the innermost band b_1 is $r_s^{\min}$ and the width of the outermost band b_n is $r_s^{\max}$ as shown in Figure 3. The idea here is to decompose our heterogeneous configuration into multiple bands, each of which has a homogeneous configuration, and leverage our previous results [8] to achieve k-coverage of each band for collectively achieving k-coverage of a PFoI using heterogeneous sensors. This hierarchical deployment of heterogeneous sensors is the solution to Question 1 stated in Section 1.2.

As can be seen, our hierarchical (or multitier) sensor deployment yields a pseudo-random sensor deployment. In fact, the sensors are deployed uniformly and randomly in each band, while any pair of square bands has different sets of sensors, thus ensuring their heterogeneity. Precisely, all the sensors present in the square band b_x are homogeneous and have the same sensing radius $r_{s,x}$. The latter is equal to the width of the square band b_x, denoted by $w(b_x)$, which is computed as follows:

$$w\,(b_x) = r_{s,\ x} = r_s^{\min} + (x-1)\,w_d, \tag{4}$$

where x is a natural number ($1 \le x \le n$), $r_s^{\min}$ is the smallest sensing radius among all the sensors, w_d is the difference of widths between any two adjacent bands, and n is the number of concentric square bands of a PFoI. Based on our hierarchical sensor deployment architecture shown in Figure 3, Lemma 3 below computes the side length of the square band b_x.

LEMMA 3 (SIDE LENGTH OF SQUARE BAND). *The side length L_x of the square band b_x is computed as follows:*

$$L_x = x\left(r_s^{\min} + r_{s,\ x}\right)$$

where x is a natural number ($1 \leq x \leq n$), $r_s{}^{min}$ is the smallest sensing radius among all the sensors, $r_{s,x}$ is the sensing radius of the sensors located in the square band b_x, and n is the number of concentric square bands of a PFoI.

PROOF. As per our hierarchical sensor deployment, which is shown in Figure 3, it is clear that half the side length of the square band b_x is the summation of the widths of the square bands b_1,b$_2$,...,b_x. Therefore, we can write

$$\frac{L_x}{2} = w(b_1) + w(b_2) + w(b_3) + \cdots + w(b_x) \Rightarrow \frac{L_x}{2} = r_s^{\min} + \left[r_s^{\min} + w_d\right]$$
$$+ \left[r_s^{\min} + 2w_d\right] + \cdots + \left[r_s^{\min} + (x-1)\,w_d\right] \qquad \text{(from Equation (4) above)}$$

This is simply the summation of x terms of an arithmetic progression (or sequence) whose initial term is $N_0 = r_s{}^{\min}$ and common difference is $d = w_d$. Hence, we have

$$\frac{L_x}{2} = \frac{x\left(r_s^{\min} + \left[r_s^{\min} + (x-1)\,w_d\right]\right)}{2} = \frac{x\left(r_s^{\min} + r_{s,\ x}\right)}{2}.$$

Therefore, we get $L_x = x(r_s^{\min} + r_{s,\ x})$. □

Leveraging the results of Lemma 3, Lemma 4 below computes the number of concentric square bands n into which a PFoI is decomposed.

LEMMA 4 (NUMBER OF CONCENTRIC SQUARE BANDS). *The number of concentric square bands n, into which a square PFoI of side length L is sliced (or decomposed), is given by*

$$n = \frac{L}{r_s^{\min} + r_s^{\max}},$$

where $r_s{}^{min}$ is the smallest sensing radius and $r_s{}^{max}$ is the largest sensing radius among all the deployed heterogeneous sensors.

PROOF. As per our hierarchical deployment, which is shown in Figure 3, it is clear that the side length of the square band b_n is L (i.e., side length of a square PFoI). Therefore, from Lemma 3, the side length L of a square PFoI can be computed as follows:

$$L = L_n = n\left(r_s^{\min} + r_s^{n}\right)$$
$$\Rightarrow L = n\left(r_s^{\min} + w(b_n)\right) \qquad \text{(from Equation (4))}$$
$$\Rightarrow L = n\left(r_s^{\min} + r_s^{\max}\right)$$
$$\Rightarrow n = \frac{L}{r_s^{\min} + r_s^{\max}}.$$

□

Lemma 5 below computes the difference between the widths w_d of any two adjacent square bands of a square PFoI.

Lemma 5 (Common Width Difference). *The common width difference, denoted by w_d, between any pair of adjacent square bands of a square PFoI, is calculated as follows:*

$$w_d = \frac{r_s^{\max} - r_s^{\min}}{n-1},$$

where n is the number of concentric square bands, ${r_s}^{min}$ is the smallest sensing radius, and ${r_s}^{max}$ is the largest sensing radius among all the sensors.

Proof. As per our hierarchical sensor deployment, which is shown in Figure 3, the width of the band b_n is ${r_s}^{\max}$. But, from Equation (4), we can compute the width of the band b_n as follows:

$$\begin{aligned} w\left(b_n\right) &= r_s^{\min} + (n-1)\, w_d \\ \Rightarrow r_s^{\max} &= r_s^{\min} + (n-1)\, w_d \\ \Rightarrow w_d &= \frac{r_s^{\max} - r_s^{\min}}{n-1}. \end{aligned}$$

□

In order to compute the planar sensor density required to k-cover a square PFoI using heterogeneous sensors, we should determine the number of tiles that are required to tessellate each square band of the square PFoI. Leveraging Equation (4) and the results of Lemma 3, Lemma 6 below estimates the number of tiles that constitute the square band b_x of the square PFoI.

Lemma 6 (Number of Tiles Per Band). *The number of tiles $t(b_x)$, which constitute the tessellation of any square band b_x of a square PFoI, can be estimated as follows:*

$$t\left(b_x\right) = 4\left[(x-1) + \frac{r_s^{\min} x}{r_s^{\min} - w_d + w_d x}\right],$$

where x is a natural number ($1 \leq x \leq n$), w_d is the difference between the widths of any two adjacent square bands, and ${r_s}^{min}$ is the smallest sensing radius among all the heterogeneous sensors.

Proof. Let us consider a square band b_x of side length L_x and whose width $w(b_x)$ is $r_{s,x}$ (from Equation (4)). From our proposed hierarchical deployment of heterogeneous sensors, we know that each band contains homogeneous sensors of sensing radius $r_{s,x}$ and any two bands have different sets of sensors. Now, for achieving k-coverage of each band, we leverage our previous results [8], where the area will be tessellated using square tiles of side length equal to the sensing radius of the sensors deployed in that area (from Section 4.1). Thus, we can estimate the number of tiles per tessellation of a square band b_x as follows:

$$\begin{aligned} t\left(b_x\right) &= \frac{\text{Area}\left(b_x\right)}{\text{Area}\left(\text{tile}\right)} \\ \Rightarrow t\left(b_x\right) &= \frac{L_x^2 - L_{x-1}^2}{\left(r_{s,\,x}\right)^2} \\ \Rightarrow t\left(b_x\right) &= \frac{\left(L_x + L_{x-1}\right)\left(L_x - L_{x-1}\right)}{\left(r_{s,\,x}\right)^2}. \end{aligned}$$

From Lemma 3, we compute $L_x + L_{x-1}$ as follows:

$$L_x + L_{x-1} = x\left(r_s^{\min} + r_{s,\ x}\right) + (x-1)\left(r_s^{\min} + r_{s,\ x-1}\right)$$

$$\Rightarrow L_x + L_{x-1} = x\left(r_s^{\min} + r_{s,\ x}\right) + (x-1)\left(r_s^{\min} + r_{s,\ x} - w_d\right) \ \left(\text{but } r_{s,\ x-1} = r_{s,\ x} - w_d\right)$$

$$\Rightarrow L_x + L_{x-1} = (2x-1)\, r_s^{\min} + (2x-1)\, r_{s,\ x} - (x-1)\, w_d.$$

From Equation (4), we have $r_{s,\ x} = r_s^{\min} + (x-1)w_d$,

$$L_x + L_{x-1} = (2x-1)\, r_s^{\min} + (2x-1)\left[r_s^{\min} + (x-1)\, w_d\right] - (x-1)\, w_d$$

$$\Rightarrow L_x + L_{x-1} = 2\left[(2x-1)\, r_s^{\min} + (x-1)^2 w_d\right]$$

$$\Rightarrow L_x + L_{x-1} = 2\left[x r_s^{\min} + (x-1)\, r_{s,\ x}\right].$$

Now, we compute $L_x - L_{x-1}$ as follows:

$$L_x - L_{x-1} = x\left(r_s^{\min} + r_{s,\ x}\right) - (x-1)\left(r_s^{\min} + r_{s,\ x-1}\right)$$

$$\Rightarrow L_x - L_{x-1} = x\left(r_s^{\min} + r_{s,\ x}\right) - (x-1)\left(r_s^{\min} + r_{s,\ x} - w_d\right) \ \left(\text{but } r_{s,\ x-1} = r_{s,\ x} - w_d\right)$$

$$\Rightarrow L_x - L_{x-1} = r_s^{\min} + r_{s,\ x} + (x-1)\, w_d$$

$$\Rightarrow L_x - L_{x-1} = 2 r_{s,\ x}$$

Therefore, we compute $t(b_x)$ as follows:

$$t\left(b_x\right) = \frac{2\left[x r_s^{\min} + (x-1)\, r_{s,\ x}\right] \times 2 r_{s,\ x}}{\left(r_{s,\ x}\right)^2}$$

$$\Rightarrow t\left(b_x\right) = \frac{4\left[x r_s^{\min} + (x-1)\, r_{s,\ x}\right]}{r_{s,\ x}}$$

$$\Rightarrow t\left(b_x\right) = 4\left[(x-1) + \frac{x r_s^{\min}}{r_{s,\ x}}\right]$$

$$\Rightarrow t\left(b_x\right) = 4\left[(x-1) + \frac{x r_s^{\min}}{r_s^{\min} + (x-1)\, w_d}\right] \quad \text{(from Equation (4))}$$

$$\Rightarrow t\left(b_x\right) = 4\left[(x-1) + \frac{x r_s^{\min}}{r_s^{\min} - w_d + x w_d}\right].$$

□

Lemma 7 below estimates the total number of tiles T of tessellations of all square bands of the PFoI, while leveraging the results of Lemma 6.

Lemma 7 (Total Number of Tiles). *The total number of tiles, denoted by T, of tessellations of all square bands of a PFoI can be estimated as follows:*

$$T = 2n(n-1) + \frac{4 n r_s^{\min}}{w_d} - \frac{4 r_s^{\min}\left(r_s^{\min} - w_d\right)}{\left(w_d\right)^2} \ln\left(\frac{2 r_s^{\min} + (2n-1)\, w_d}{2 r_s^{\min} - w_d}\right),$$

where n is the number of concentric square bands, w_d is the difference between the widths of any two adjacent square bands, and $r_s{}^{min}$ is the smallest sensing radius among all the heterogeneous sensors.

PROOF. From Lemma 6, we have the number of tiles $t(b_x)$ per square band b_x computed as follows:

$$t\left(b_x\right) = 4\left[\left(x-1\right) + \frac{r_s^{\min} x}{r_s^{\min} - w_d + w_d x}\right].$$

Thus, we can compute the total number of tiles T of tessellations of all square bands as follows:

$$\begin{aligned}
&T = \sum_{x=1}^{n} t\left(b_x\right)\\
&\Rightarrow T = \sum_{x=1}^{n} 4\left[\left(x-1\right) + \frac{r_s^{\min} x}{r_s^{\min} - w_d + w_d x}\right]\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + 4r_s^{\min}\sum_{x=1}^{n}\frac{x}{r_s^{\min} - w_d + w_d x}\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + \frac{4r_s^{\min}}{w_d}\sum_{x=1}^{n}\frac{w_d x}{r_s^{\min} - w_d + w_d x}\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + \frac{4r_s^{\min}}{w_d}\sum_{x=1}^{n}\frac{\left(r_s^{\min} - w_d + w_d x\right) - \left(r_s^{\min} - w_d\right)}{r_s^{\min} - w_d + w_d x}\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + \frac{4r_s^{\min}}{w_d}\sum_{x=1}^{n}\left[1 - \frac{r_s^{\min} - w_d}{r_s^{\min} - w_d + w_d x}\right]\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + \frac{4r_s^{\min}}{w_d}\sum_{x=1}^{n}1 - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{w_d}\sum_{x=1}^{n}\frac{1}{r_s^{\min} - w_d + w_d x}\\
&\Rightarrow T = 4\sum_{x=1}^{n}\left(x-1\right) + \frac{4r_s^{\min}}{w_d}\sum_{x=1}^{n}1 - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{w_d}\sum_{x=1}^{n}\frac{1}{r_s^{\min} + \left(x-1\right)w_d}.
\end{aligned}$$

We have

$$\begin{aligned}
&\sum_{x=1}^{n}\left(x-1\right) = \frac{n\left(n-1\right)}{2},\\
&\sum_{x=1}^{n}1 = n,\\
&\sum_{x=1}^{n}\frac{1}{a + \left(x-1\right)d} = \frac{1}{d}\ln\left(\frac{2a + \left(2n-1\right)d}{2a - d}\right)\\
&\Rightarrow T = 4 \times \frac{n\left(n-1\right)}{2} + \frac{4r_s^{\min}}{w_d}\times n - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{w_d}\times\frac{1}{w_d}\ln\left(\frac{2r_s^{\min} + \left(2n-1\right)w_d}{2r_s^{\min} - w_d}\right)\\
&\Rightarrow T = 2n\left(n-1\right) + \frac{4nr_s^{\min}}{w_d} - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{\left(w_d\right)^2}\ln\left(\frac{2r_s^{\min} + \left(2n-1\right)w_d}{2r_s^{\min} - w_d}\right).
\end{aligned}$$

□

Leveraging the results of Lemma 7, Theorem 2 below computes the planar sensor density that is required to k-cover a PFoI using our hierarchical deployment of heterogeneous sensors. This planar sensor density computed by Theorem 2 is the solution to Question 2 stated in Section 1.2.

Theorem 2 (Planar Sensor Density). *The planar sensor density λ, which is required to k-cover a PFoI using our hierarchical deployment of heterogeneous sensors, is computed as follows:*

$$\lambda = \frac{k}{L^2}\left[2n\left(n-1\right) + \frac{4nr_s^{\min}}{w_d} - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{\left(w_d\right)^2}\ln\left(\frac{2r_s^{\min} + \left(2n-1\right)w_d}{2r_s^{\min} - w_d}\right)\right],$$

where k is the degree of coverage, L is the side length of a square PFoI, n is the number of concentric square bands, w_d is the difference between the widths of any two adjacent square bands, and ${r_s}^{min}$ is the smallest sensing radius among all the heterogeneous sensors.

Proof. From Definition 5 (Section 3.1), the planar sensor density is the number of heterogeneous sensors required per unit area to k-cover a PFoI. Therefore, the planar sensor density λ can be computed as follows:

$$\lambda = \frac{\text{Total number of sensors per } k - \text{coverage round}}{\text{Area of the PFoI}}$$
$$\Rightarrow \lambda = \frac{k \times T}{\text{Area of the PFoI}}.$$

□

Leveraging the results of Lemma 7, we have

$$\Rightarrow \lambda = \frac{k}{L^2}\left[2n\left(n-1\right) + \frac{4nr_s^{\min}}{w_d} - \frac{4r_s^{\min}\left(r_s^{\min} - w_d\right)}{\left(w_d\right)^2}\ln\left(\frac{2r_s^{\min} + \left(2n-1\right)w_d}{2r_s^{\min} - w_d}\right)\right].$$

Lemma 8 below states the relationship that should exist between the sensing radius $r_{s,x}$ and the communication radius $r_{c,x}$ of the sensors deployed in a square band b_x for ensuring network connectivity of all heterogeneous sensors deployed in various concentric square bands of a PFoI. This type of relationship is essential for attaining connected k-coverage during the whole operational lifetime of heterogeneous PWSNs. This relationship established by Lemma 8 is the solution to Question 3 in Section 1.2.

Lemma 8 (Network Connectivity). *Based on our hierarchical deployment of heterogeneous sensors, a square tessellation-based k-coverage configuration is said to be connected if the sensing radius $r_{s,x}$ and the communication radius $r_{c,x}$ of the sensors deployed in a square band b_x satisfy the following inequality:*

$$r_{c,\ x} \geq 2r_{s,\ x} + w_d,$$

where w_d is the difference between the widths of any two adjacent square bands.

Proof. The necessary condition for ensuring network connectivity of any two adjacent square bands is that the two farthest sensors (one from the first band and the other one from the second band) should be able to communicate with each other. Let us consider two adjacent concentric square bands, say b_x and b_{x+1}, of our hierarchical heterogeneous sensor deployment. As shown in Figure 4, a sensor s_x of tile T_2 of band b_x is the farthest from a sensor s_{x+1} of tile T_1 of band b_{x+1} if sensor s_x is present on the vertex E_2 of tile T_2 and sensor s_{x+1} is present on the vertex G_1 of tile T_1. Therefore, sensor s_x of band b_x will be able to communicate with sensor s_{x+1} of band b_{x+1} if the communication radius $r_{c,x}$ of sensor s_x is at least equal to the distance between vertex G_1 of tile T_1 and vertex E_2 of tile T_2. From Lemma 1, we know that the distance between a vertex of square tile from the farthest vertex of its cusp-square area is equal to the side length of the square tile.

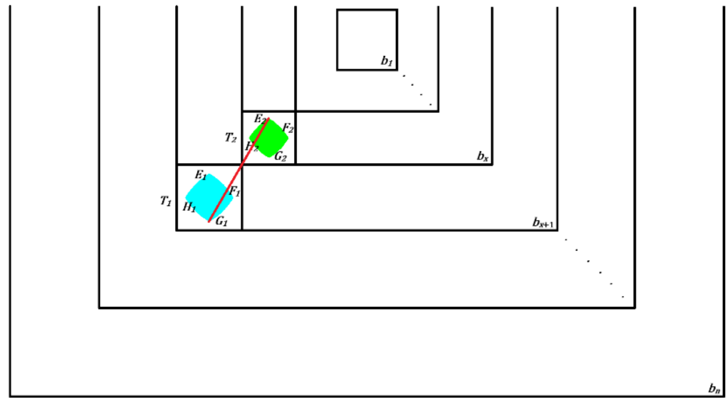


Fig. 4. Adjacent concentric square bands b_x and b_{x+1}.

Therefore, we have

$$\begin{aligned} & r_{c,\ x} \geq \left|\overline{G_1E_2}\right| \\ & \Rightarrow r_{c,\ x} \geq r_{s,\ x} + r_{s,\ x+1} \\ & \Rightarrow r_{c,\ x} \geq r_{s,\ x} + r_{s,\ x} + w_d \ \left(\text{but } r_{s,\ x+1} = r_{s,\ x} + w_d\right) \\ & \Rightarrow r_{c,\ x} \geq 2r_{s,\ \text{x}} + w_d \end{aligned}$$

□

Leveraging all the above discussed and proved mathematical results and properties, we introduce our heterogeneous connected k-coverage protocol, called Het-k-CSqu, which utilizes our hierarchical deployment of heterogeneous sensors as well as cusp-square areas of the square tiles of the tessellation [8] for each square band of a PFoI. Going forward, in the next sections of the article, we use the terms "connected k-coverage" and "k-coverage" interchangeably. Next, we will discuss Het-k-CSqu in detail

5 Centralized Heterogeneous *k*-Coverage Protocol

In this section, we discuss our centralized *Het*erogeneous k-coverage protocol using *C*usp *Squ*ares, denoted by *Het-k-CSqu*, which is performed by the sink to ensure k-coverage of a PFoI using heterogeneous sensors. Our protocol has two phases: The first phase (Section 5.1) generates cusp-square based square tessellation of each band, whereas the second phase (Section 5.2) is responsible for the selection and scheduling of the sensors. This constitutes our solution to Question 4 stated in Section 1.2. Next, we discuss both phases in detail.

5.1 Generation of Square Tessellation for Bands

In this phase, before starting the k-coverage round, the sink tessellates each concentric square band of a PFoI using square tiles. Then, as discussed in Section 4.1, it constructs cusp-square areas for all square tiles of the tessellation of every square band of a PFoI. Later, for every square band, the sensors are scheduled for each k-coverage round. Furthermore, for all square tiles of all tessellations of all square bands, the generated cusp-square areas remain static (unchanged) throughout all the k-coverage rounds. Moreover, all these cusp-square areas act as restriction areas for the sensor selection process as well as for providing movement instructions to the sensors, across all tessellations of square bands in order to k-cover a PFoI collectively.

ALGORITHM 1: Het-k-CSqu

Begin

1. $n = L/(r_s^{\max} + r_s^{\min})$ /*Compute n using Lemma 4*/

2. $w_d = (r_s^{\max} + r_s^{\min})/(n-1)$ /* Compute w_d using Lemma 5*/

3. *sens_rads = Determine_Heterogeneous_Sensing_Ranges(n, w_d, $r_s^{\max}$, $r_s^{\min}$)*
/* *Returns a list of all sensing radii of deployed heterogeneous sensors using Equation 4**/

4. *Sq_Bands = Decompose_Field_of_Interest(n, w_d, $r_s^{\max}$, $r_s^{\min}$)* /* *Returns a list of square bands* */
/* *NOTE: sens_rads[i] is the sensing radius of sensors deployed in Sq_Bands[i]* */

5. *kCov_Sensors* = { } /* *Empty set* */

6. **For** *index* = 1 **to** *Length(Sq_Bands)* **Do**

Begin

6.1. *Sq_Tiles = Tessellate_Band(Sq_Bands[index], sens_rads[index])*
/* *Generate cusp-square based square tessellation for the band using the sensing radius and returns list of tiles* */

6.2. **For** *Tile* **in** *Sq_Tiles* **Do**

Begin

6.2.1. Select subset S_{Tile} of sensors location on the *Tile.*

6.2.2. $j = 1$

6.2.3. **While** $j \leq k$ **Do** /* *k is expected degree of coverage* */

Begin

6.2.3.1. Select one sensor s from S_{Tile} with highest remaining energy

6.2.3.2. **If** *Location(s)* is outside of *Cusp_Square(Tile)*

Then

6.2.3.2.1. *Add_Movement_Instructions(s)*

6.2.3.3. $j = j + 1$

6.2.3.4. Add sensor s to *kCov_Sensors* set.

End

End

End

7. **Return** *kCov_Sensors*

End

5.2 Sensor Selection and Duty Cycling

During this phase, the sink chooses the sensors and arranges for the participants in the k-coverage rounds. The primary objective of this phase is to select and duty cycle (or schedule) the sensors in a way that ensures almost a constant energy depletion rate for all the k-coverage rounds, such that all sensors have comparable lifetimes. The secondary objective of this phase is to achieve optimal battery-power consumption per k-coverage round by all active sensors, implicitly ensuring a prolonged lifetime of the individual sensors and the lifetime of the whole network. Also, the primary and secondary objectives are interrelated, i.e., achieving either objective will ensure the other.

The sink assigns a unique identifier (*id*) to each sensor. The selection and duty cycling of the sensors for each k-coverage round is prioritized based on the location and remaining battery power of the individual sensor. Initially all the sensors will be in the *sleep* mode, and switch to the *awake* mode for receiving the k-coverage schedule from the sink, just before the start of every k-coverage round. Usually the k-coverage schedule is a list of sensors that are selected for that specific round. The sink broadcasts this k-coverage schedule at the end of each k-coverage round. Every sensor checks for its own *id* in the k-coverage schedule. If a sensor finds its *id* in this schedule, it will remove its *id* from the schedule, share the updated k-coverage schedule with its one-hop neighbors, and switch to the *active* mode for k-covering the area. Once the sensor switches to *active* mode, it checks for any additional movement instructions provided by the sink and moves accordingly. If the schedule does not contain the sensor's *id*, the sensor will simply share the received k-coverage schedule with its one-hop neighbors and switches to the *sleep* mode for energy conservation. Occasionally, the selected sensors may be located outside the cusp-square areas of the square tiles. In this case, the sink will determine the best possible location within the cusp-square area of the tile, corresponding to the sensor's current location and adds this new location as additional movement instructions, such that the sensor moves to this new location ensuring the sensor within the cusp-square area.

6 Performance Evaluation

In this section, we discuss the performance results of our proposed heterogeneous k-coverage protocol, Het-k-CSqu, and compare these results with an existing heterogeneous k-coverage protocol. We leveraged an open- source high-level PWSN simulator by Darolt, developed in *Python* and *C++* programming languages, for simulating network operations. We updated the network component of the simulator in order to model our tessellation-based k-coverage theory by accepting the tile shape and the degree of coverage k as inputs. Apart from existing energy plot functionality, we updated the simulator's plot functionalities for the experimental results discussed in the following subsequent sections. Next, we briefly discuss our simulation environment and its parameters, and then present the simulation results of our proposed protocol for solving the connected k-coverage problem in heterogeneous PWSNs.

6.1 Simulation Environment

As discussed in Section 4.3, we have considered a square-shaped PFoI for all the experimental scenarios. As previously discussed in Section 3.3, our energy model considers all types of battery-power consumption, including data sensing, data transmission, data reception, sensor mobility, and control messages, for ensuring the correct operation of our heterogeneous k-coverage protocol, Het-k-CSqu. We have leveraged the IEEE 802.11 distributed coordinated function with CSMA/CA as the underlying MAC protocol. Furthermore, we considered the radio interference model given the pervasiveness of other 2.4 GHz radio sources. All the simulations are performed on a 10th Gen Intel (R) Core (TM) i7-10750 2.60 GHz CPU with 16 GB of RAM using a 64-bit Windows 11 operating system environment. The network parameters used for the simulations are presented in Table 1 below.

6.2 Simulation Results

In this section, we present the simulation results of our Het-k-CSqu protocol in comparison with the result in Theorem 2 of Section 4.3 and homogeneous protocol k-CSqu [8]. For having a fair comparison of our homogeneous protocol k-CSqu [8] with our heterogeneous protocol Het-k-CSqu, we assumed a sensing radius r_s = 45 m (i.e., mean value of the sensing radius of the heterogeneous sensors given in Table 1) and E_{init} = 70 J (i.e., mean value of the initial battery power of the heterogeneous sensors given in Table 1) for protocol k-CSqu [8].

Figure 5 shows the variation of the planar sensor density λ with respect to the degree of coverage k. As expected, the planar sensor density λ is directly proportional to the degree of coverage k because the planar sensor density increases with the increase in the degree of coverage. From the plot, it is clear that we were able to achieve a simulation planar sensor density for Het-k-CSqu that is near the theoretical one calculated by Theorem 2. This indicates that we were able to implement our hierarchical deployment of heterogeneous sensors and square tessellation-based

Table 1. Network Parameters of the Simulations

Parameter	Description	Value
L	Side length of the Field of Interest	990 m
N	Total number of sensors deployed	4000
α	Path-loss exponent	[2, 4]
ε_{fs}	Transmitter amplifier in free space	10 pJ/bit/m^2
ε_{mp}	Transmitter amplifier in multipath	0.0013 pJ/bit/m^4
E_{init}	Initial battery power of heterogeneous sensors	55–85 J
E_e	Electronic energy consumption	50 nJ/bit
E_{move}	Energy consumed by sensor per distance moved	0.008–0.012 J/m
E_{idle}	Energy consumed by sensor in idle mode	0.012 J
E_{sleep}	Energy consumed by sensor in sleep mode	0.0003 J
r_s	Sensing radius of heterogeneous sensors	30–60 m
r_c	Communication radius of heterogeneous sensors	65–125 m
k	Degree of coverage	3

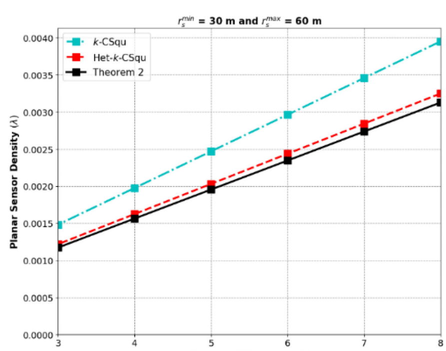


Fig. 5. Planar sensor density λ *vs.* degree of coverage k.

k-coverage for all the square bands with a highest accuracy and have a close match with our theory developed in Section 4.3. Moreover, our heterogeneous protocol Het-k-CSqu outperforms our homogeneous protocol k-CSqu [8] in terms of the planar sensor density for every achievable degree of coverage. This indicates that Het-k-CSqu requires a smaller number of active sensors for achieving k-coverage of a PFoI compared to k-CSqu [8]. This is a fair achievement by Het-k-CSqu given the heterogeneity of the sensors and their varied characteristics in terms of their sensing range, communication range, and initial battery power, along with the hierarchical deployment of heterogeneous sensors.

Figure 6 shows the number of active sensors n_a that is required to k-cover the PFoI, compared to the number of deployed sensors n_d. Figure 6(a) results support our previous inference that Het-k-CSqu achieves a highest accuracy in implementing our heterogeneous k-coverage theory discussed in Section 4.3. These results also support that Het-k-CSqu requires a smaller number of active sensors for achieving k-coverage of a PFoI compared to k-CSqu [8]. Figure 6(b) shows the effect of degree of coverage k on the number of active sensors n_a that is required for k-covering the PFoI. From both plots, it is clear that our square tessellation-based k-coverage theory ensures constant n_a for both homogeneous and heterogeneous PWSN configurations. Moreover, the number of active

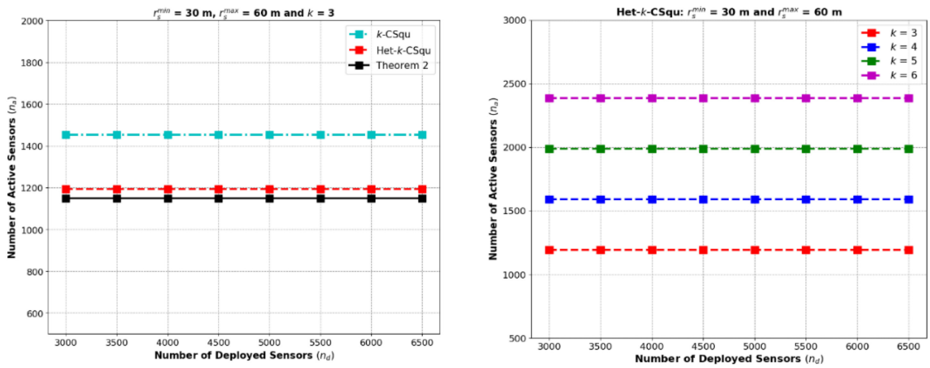


Fig. 6. Number of active sensors n_a vs. number of deployed sensors n_d: (a) Comparison and (b) Het-k-CSqu with varying k.

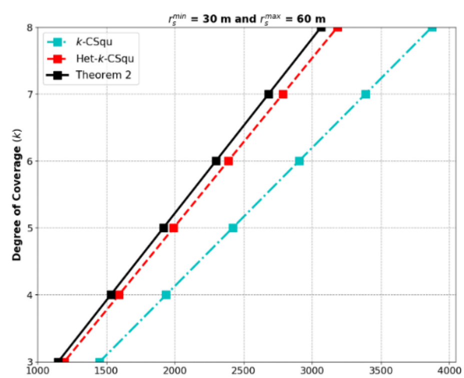


Fig. 7. Degree of coverage k vs. number of active sensors n_a.

sensors n_a required does not depend on the number of deployed sensors n_d as it only depends on the number of tiles of the tessellation.

Figure 7 plots the variation of the degree of coverage k with respect to the number of active sensors n_a. It is clear that k increases proportionally with n_a, supporting the results in Figure 5. Also, it is worth noting that for a constant number of active sensors, Het-k-CSqu achieves a higher degree of coverage compared to k-CSqu [8]. All the results of Figures 4, 5, and 6 imply that sensor heterogeneity ensures the deployment of a minimum number of active sensors for k-coverage, which in turn ensures a longer operational network lifetime.

Remark 1. A heterogeneous PWSN ensures the use of a smaller number of active sensors for k-coverage compared to a homogeneous PWSN. This helps ensure a more effective energy consumption by the sensors as well as a longer operational network lifetime.

6.3 Comparison of Het-k-CSqu with PR-Het-CCC$_k$

In this section, we compare our Het-k-CSqu with PR-Het-CCC$_k$ [31]. As discussed in Section 2.3, the pseudo-random sensor deployment [31] fails to achieve optimality in terms of the total number of tiles required to tessellate a PFoI. Hence, we implemented the PR-Het-CCC$_k$ protocol [31] using our hierarchical deployment strategy and performed the experiments. This helps provide a fair comparison between Het-k-CSqu and PR-Het-CCC$_k$ [31].

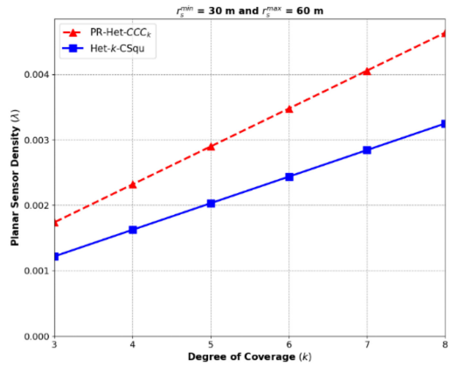


Fig. 8. Planar sensor density λ vs. degree of coverage k for Het-k-CSqu and PR-Het-CCC$_k$.

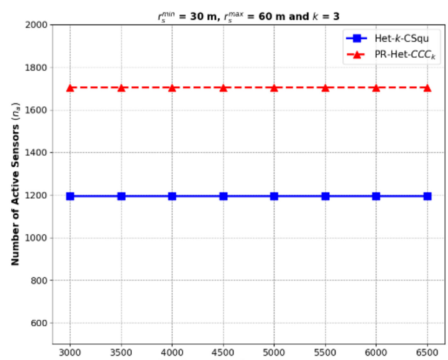


Fig. 9. Number of active sensors n_a vs. number of deployed sensors n_d for Het-k-CSqu and PR-Het-CCC$_k$.

Figure 8 presents the simulation results of the planar sensor density λ for Het-k-CSqu and PR-Het-CCC$_k$ [31] with varying degrees of coverage k. From the plot, it is clear that our protocol Het-k-CSqu requires a lower planar sensor density λ compared to PR-Het-CCC$_k$ [31] for every achievable degree of coverage k. Thus, for a desired degree of coverage k of a PFoI, our protocol Het-k-CSqu requires a smaller number of sensors compared to PR-Het-CCC$_k$ [31] for achieving k-coverage using heterogeneous sensors. Conversely, for a constant planar sensor density λ, Het-k-CSqu achieves a higher degree of coverage compared to PR-Het-CCC$_k$ [31].

Figure 9 highlights the differences between Het-k-CSqu and PR-Het-CCC$_k$ [31] in terms of the number of active sensors n_a compared to the number of deployed sensors n_d. As inferred from Figure 8 results, our protocol Het-k-CSqu requires a lesser number of active sensors n_a compared to PR-Het-CCC$_k$ [31] for achieving k-coverage of a PFoI. As PR-Het-CCC$_k$ [31] also leverages tessellation-based k-coverage theory, the number of active sensors n_a remained constant for any number of deployed sensors n_d, proving our previous deduction from Figure 6 results. This indicates that, for a given degree of coverage k and heterogeneous sensors of sensing range between $r_s{}^{\min}$ and $r_s{}^{\max}$, our protocol, leveraging our hierarchical sensor deployment and square tessellation-based k-coverage, ensures a more efficient energy consumption by the active sensors per k-coverage round, which in turn ensures a longer operational network lifetime for our protocol.

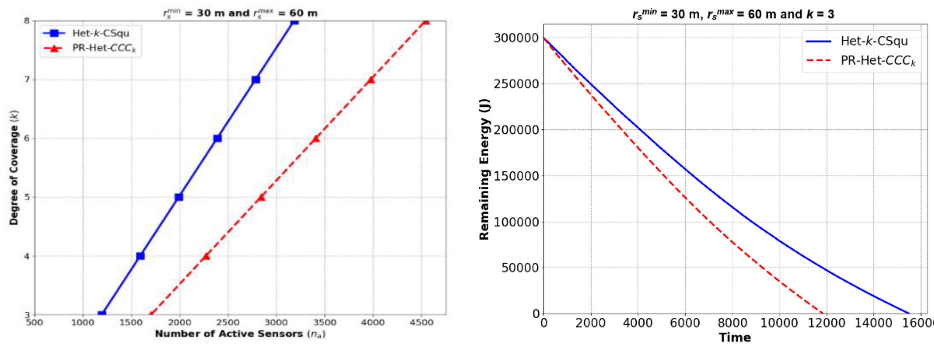


Fig. 10. Het-k-CSqu and PR-Het-$\mathrm{CCC_k}$: (a) Degree of coverage k vs. number of active sensors n_a. (b) Remaining energy vs. time.

Remark 2. A heterogeneous PWSN employing our Het-k-CSqu protocol uses a smaller number of active sensors for achieving k-coverage of a PFoI compared to a heterogeneous PWSN employing PR-Het-$\mathrm{CCC_k}$ protocol [31].

Figure 10(a) plots the degree of coverage k versus the number of active sensors n_a for Het-k-CSqu and PR-Het-CCC_k [31]. This plot is evident and proves our previous inference that for the same number of active sensors, Het-k-CSqu offers a higher degree of coverage k compared to that of PR-Het-CCC_k [31]. Therefore, for any degree of coverage k, our protocol Het-k-CSqu yields significant energy savings, which in turn helps extend the operational network lifetime. Figure 10(b) plots the remaining energy of all the heterogeneous sensors versus time in order to show the rate of energy consumption per k-coverage round and the operational network lifetime of Het-k-CSqu and PR-Het-CCC_k [31] protocols. These results prove our previous inference, and it is evident that the operational network lifetime obtained for Het-k-CSqu is higher than that of PR-Het-CCC_k [31].

Remark 3. A heterogeneous PWSN employing the Het-k-CSqu protocol has a lower energy consumption per k-coverage round compared to a heterogeneous PWSN employing the PR-Het-$\mathrm{CCC_k}$ protocol.

Remark 4. A heterogeneous PWSN employing the Het-k-CSqu protocol has a longer operational network lifetime compared to a heterogeneous PWSN employing the PR-Het-$\mathrm{CCC_k}$ protocol.

7 Conclusion

7.1 Summary

In this article, we investigate the problem of connected k-coverage in heterogeneous PWSNs using a hierarchical deployment of heterogeneous sensors and a square tessellation-based approach. Precisely, as part of the hierarchical deployment, we decompose a square PFoI into concentric square bands based on the side length L of the square PFoI, the minimum sensing radius $r_s^{\min}$, and the maximum sensing radius $r_s^{\max}$ of the deployed heterogeneous sensors. Each of these square bands has homogeneous sensors, which are uniformly and randomly deployed and whose sensing radius is equal to the width of that square band, such that any two square bands have different sets of sensors in terms of their sensing range, communication range, and initial battery power. For each square band, k-coverage is attained by leveraging cusp-square area-based square tessellation (discussed in Section 4.1). Combining hierarchical heterogeneous sensor deployment and square tessellation-based k-coverage, we compute the planar sensor density (Theorem 2 in Section 4.3) that is required to maintain k-coverage of a PFoI using heterogeneous sensors. Also, leveraging this combination, we establish a relationship that should exist between the sensing and communication radii of the sensors of any specific square band for ensuring network connectivity (Lemma 8 in Section 4.3), thus attaining connected k-coverage configurations using heterogeneous sensors during the network operation. Furthermore, based on all the proved theoretical results and properties, we propose a centralized heterogeneous connected k-coverage protocol, Het-k-CSqu. Based on the simulation results, it is evident that Het-k-CSqu has better performance and is more energy-efficient compared to PR-Het-CCC$_k$ [31], in terms of the number of active sensors (or planar sensor density) to achieve k-coverage and operational network lifetime for k-coverage of a PFoI.

7.2 Future Work

Our future work is fivefold. In this work, we assume that the sensors are deployed densely in the PFoI. First, we want to investigate the problem of connected k-coverage in heterogeneous PWSNs, where the sensors are sparsely deployed in a PFoI. Second, we want to generalize our current work to account for the irregularity of the sensing and communication capabilities of the heterogeneous sensors by considering stochastic models [12, 17] rather than deterministic models. Third, we focus on the problem of connected k-coverage in three-dimensional heterogeneous WSNs, such as underwater WSNs [40], by exploiting the concept of cube [42] (i.e., the counterpart of square in the three-dimensional Euclidean space). We want to assess the combination of sensor heterogeneity and dimensionality (three-dimensional vs. two-dimensional Euclidean space) for a more challenging connected k-coverage problem dealing with both cases, namely, homogeneous and heterogeneous sensor deployment. From Equation (4), Lemma 4, and Lemma 5 above, our proposed hierarchical sensor deployment has a limitation on the number of concentric square bands n and their width difference w_d. Both of these variables are dependent on the side length L of a PFoI as well as the minimum and maximum sensing radii of the heterogeneous sensors, namely, ${r_s}^{\min}$ and ${r_s}^{\max}$, respectively. As our fourth direction, we plan to extend our hierarchical sensor deployment to alleviate this problem by having any number of concentric square bands in a PFoI. This will enable the deployment of various types of sensors with respect to their sensing capabilities, and without any restrictions. Finally, we plan to place our protocol into practice using a real-world sensor test bed [41].

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