

# Achieving Connected $k$ -coverage in Wireless Sensor Networks using Computational Geometry-based approaches

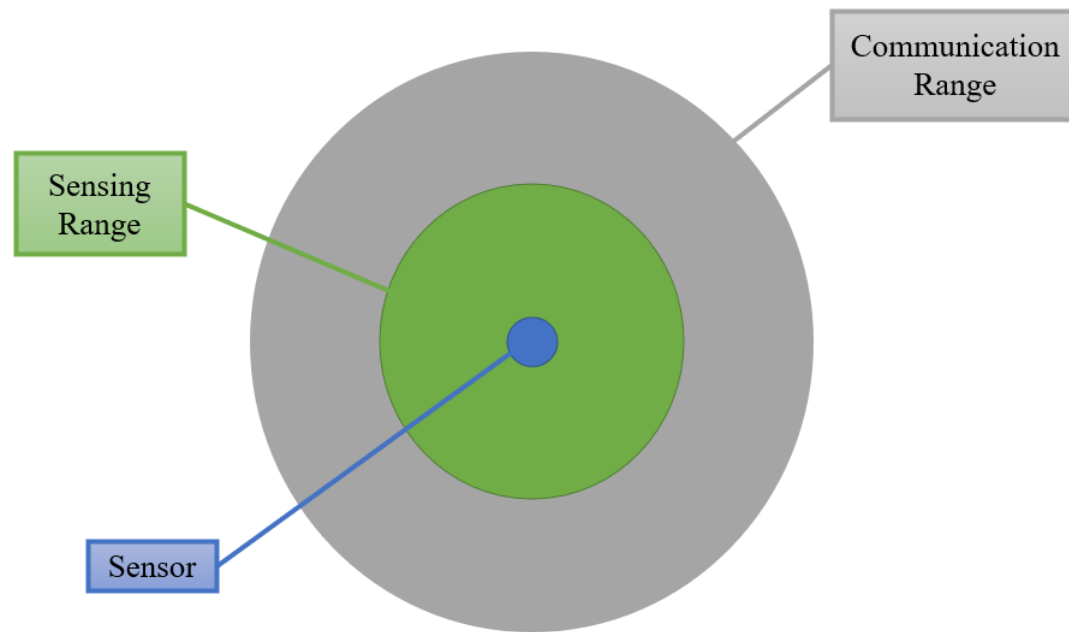
*Venkata Swamy **Kalyan** Nakka*

# Outline

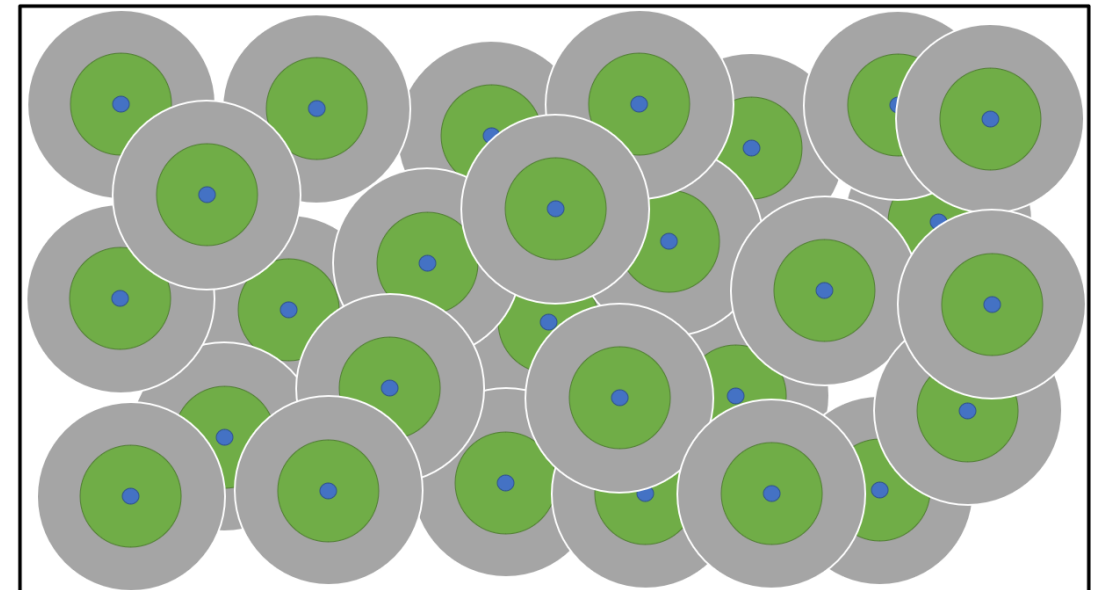
- **Introduction**
- Dr. Ammari's Research
- Preliminaries
- Square Tessellation Approach
- Hexagonal Tessellation Approach
- Results
- Conclusion

# Planar Wireless Sensor Networks

Sensor



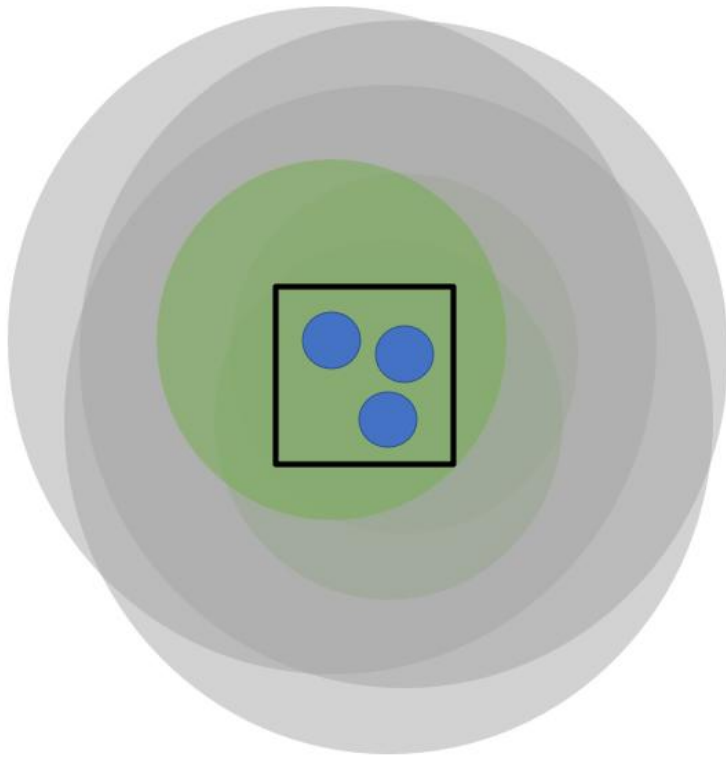
Sensors deployed in FoI



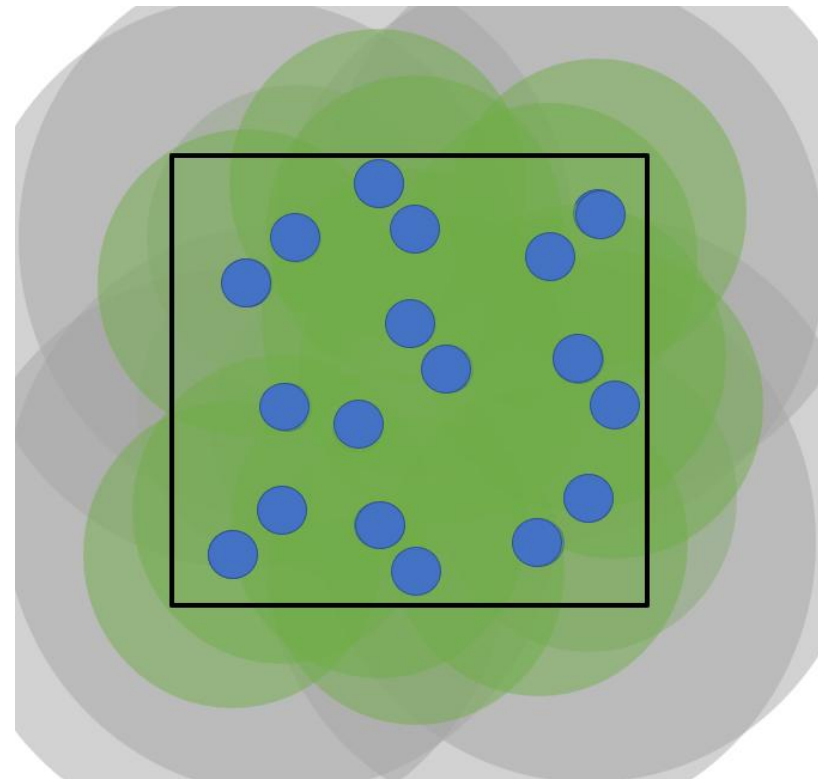
Field of Interest (FoI)

# Problem of Connected $k$ -coverage in Wireless Sensor Networks

$k$ -coverage



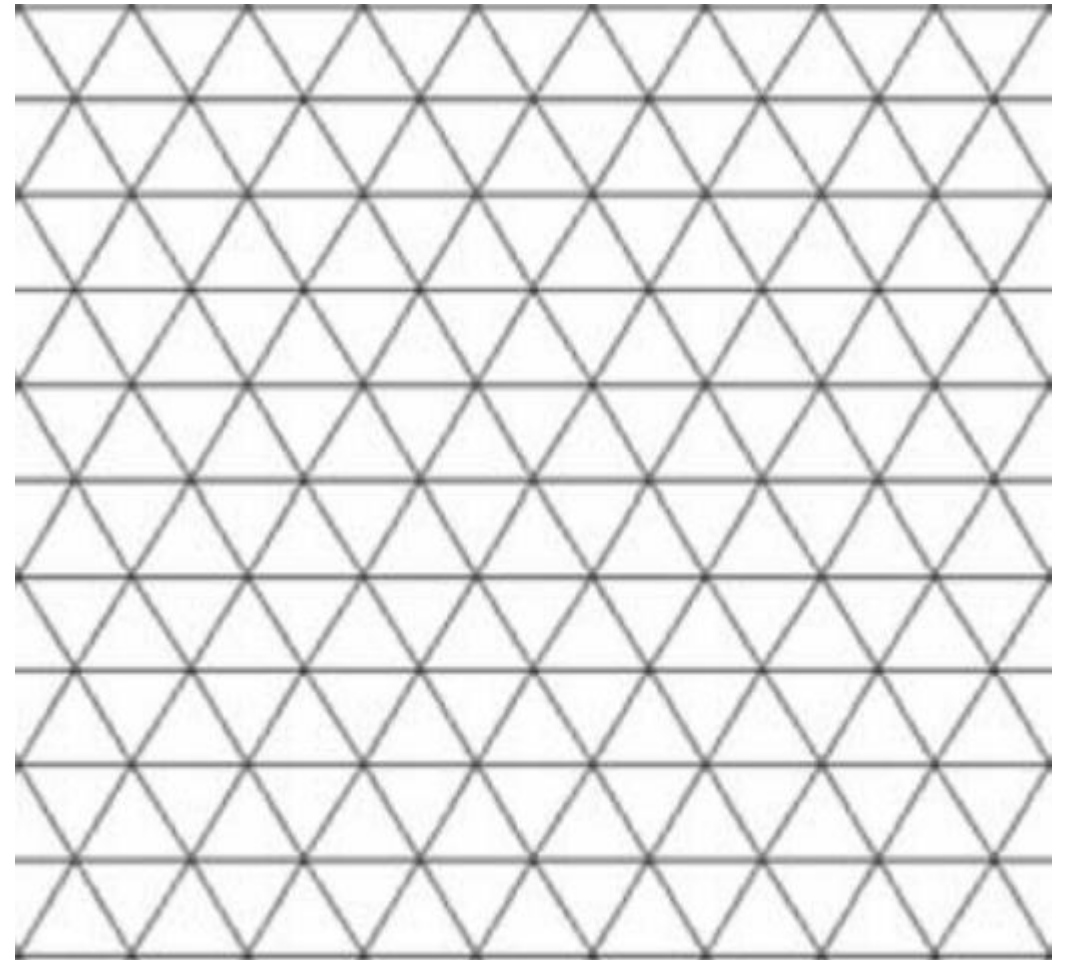
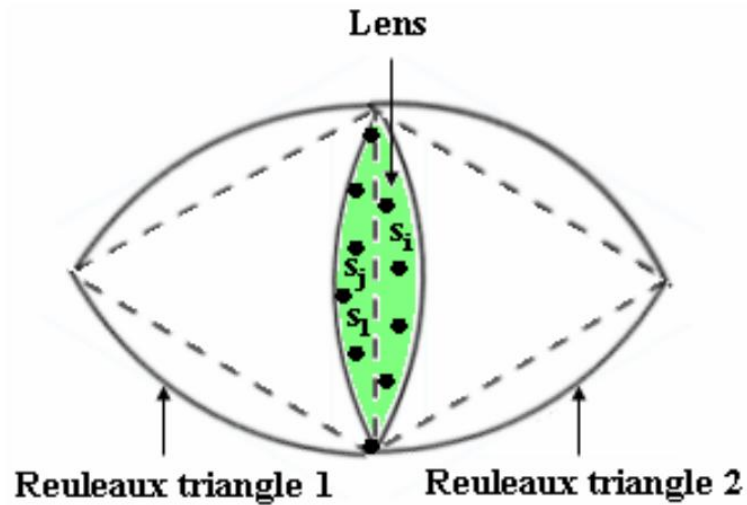
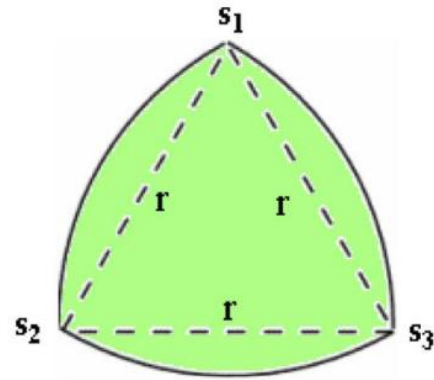
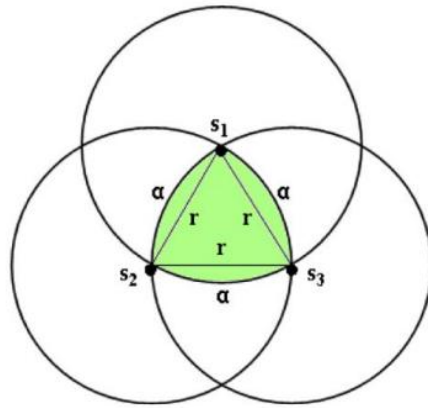
Connected  $k$ -coverage



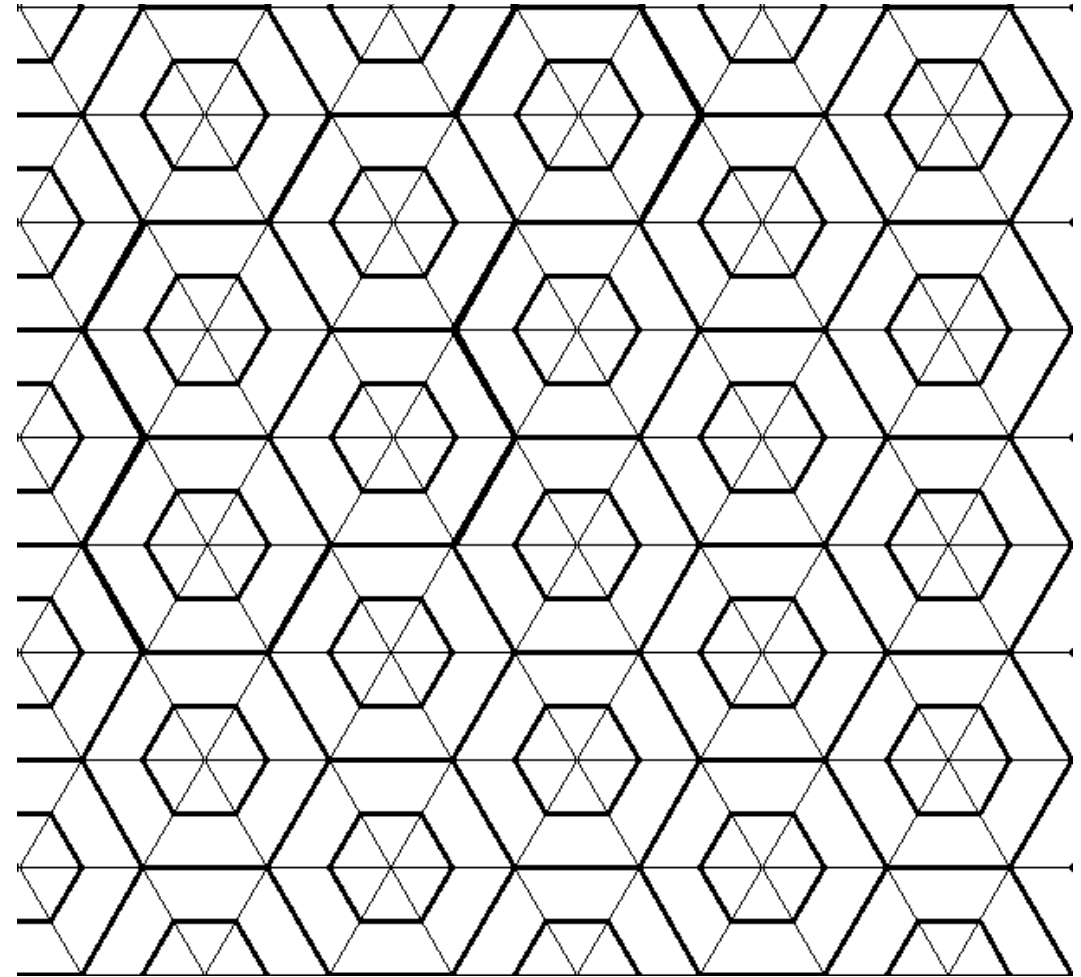
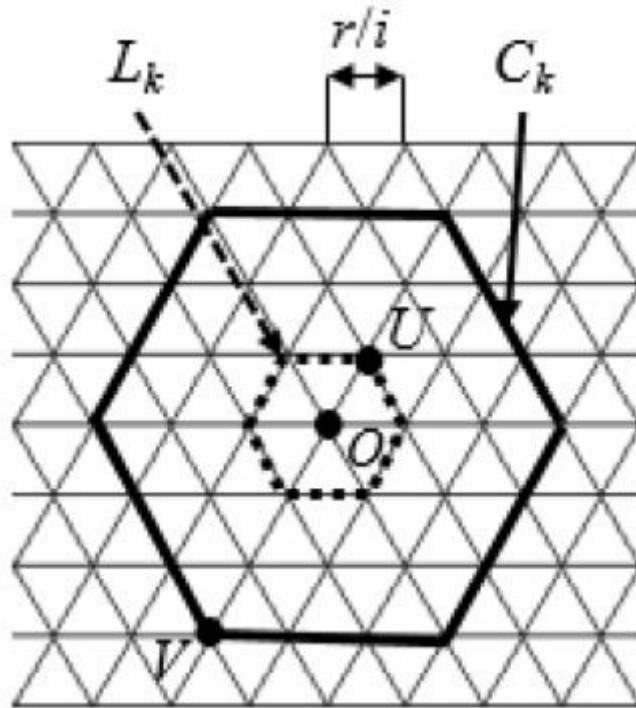
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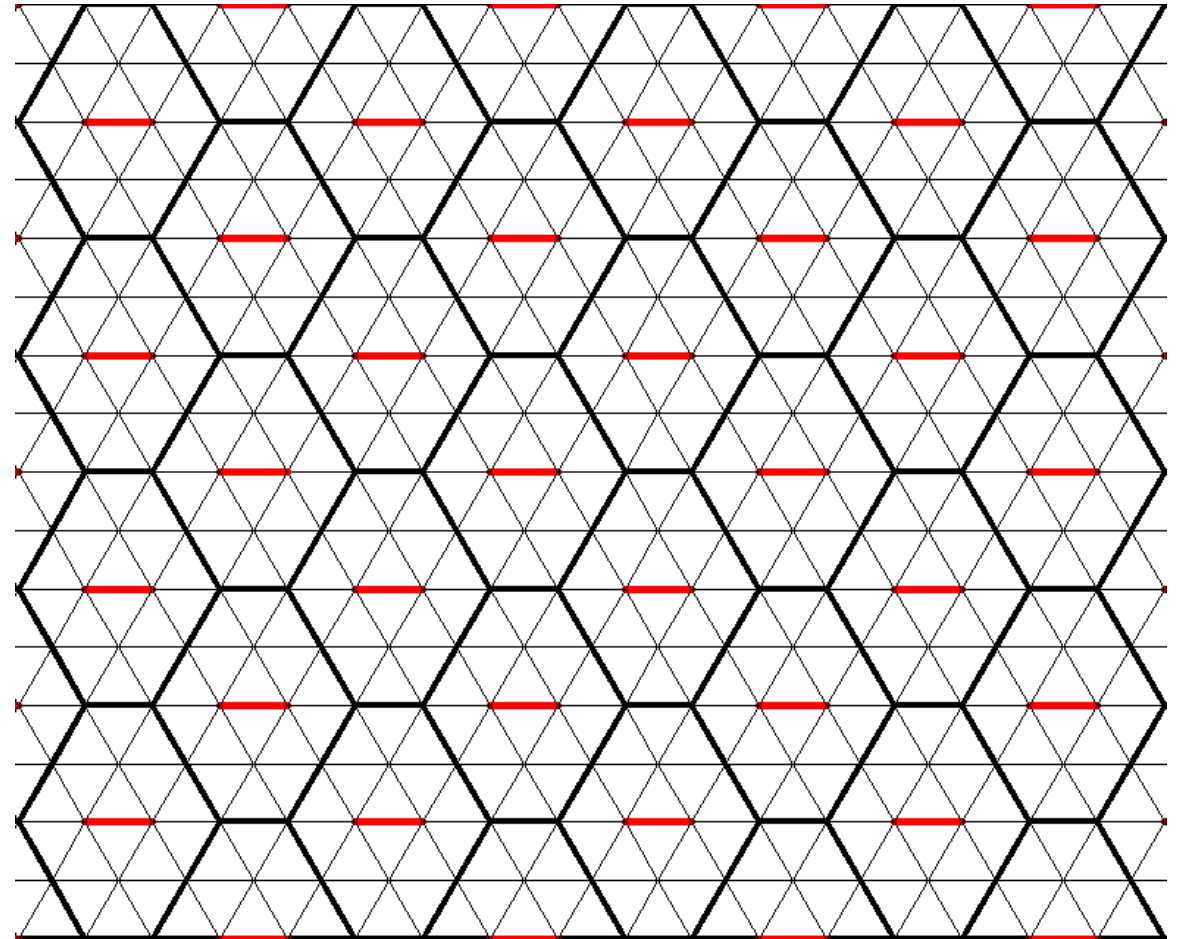
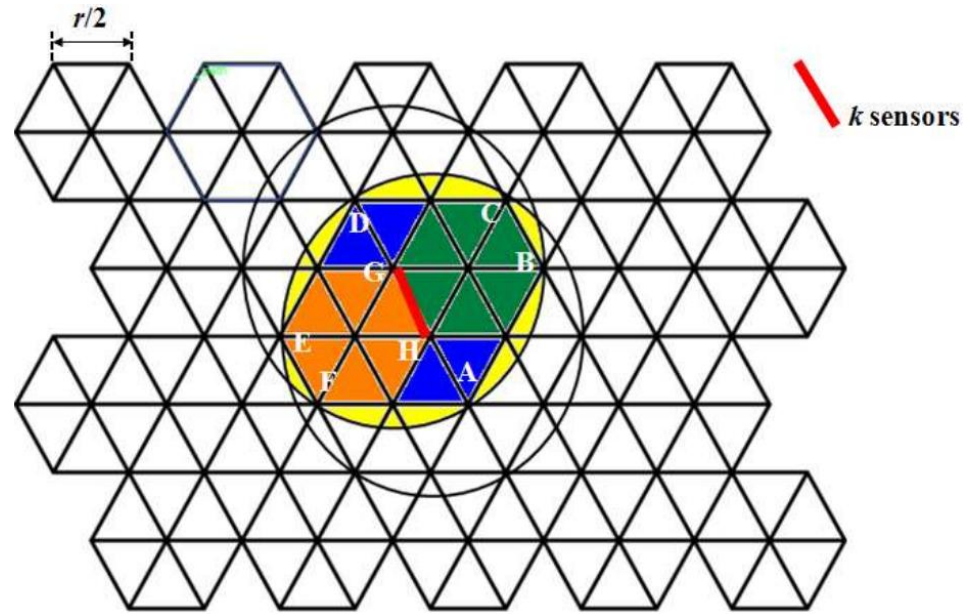
# Reuleaux Triangle Tessellation



# Regular Hexagonal Tessellation



# Irregular Hexagonal Tessellation





# How to achieve Connected $k$ -coverage ?

1. Decide a *Tile*.
2. Determine the Sensor Placement strategy for that Tile.
3. Compute the necessary relation for ensuring network connectivity in the proposed tessellation.

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# Sensing Model

- Deterministic Sensing Model
- Every point  $P$  in the field may be sensed by a sensor  $s$  if and only if the Euclidean distance  $\delta(P, s)$  is lower than or same as sensing range.
- The point  $P$  sensed by sensor  $s$  is denoted by  $Cov(P, s)$ , where,

$$Cov(P, s) = \begin{cases} 1, & \text{if } \delta(P, s) \leq r_s \\ 0, & \text{otherwise} \end{cases}$$

# Network Model

- All sensors are randomly and densely dispersed in the FoI.
- All sensors are homogeneous.
- All sensors and sink are aware of their own locations.
- Both sensing and communication ranges of all sensors are modelled to be of disk-shaped.
- All sensors are mobile and can move to specified locations.
- All sensors drain their power supplies for tasks like sending and receiving data, detecting, moving about, and other activities.

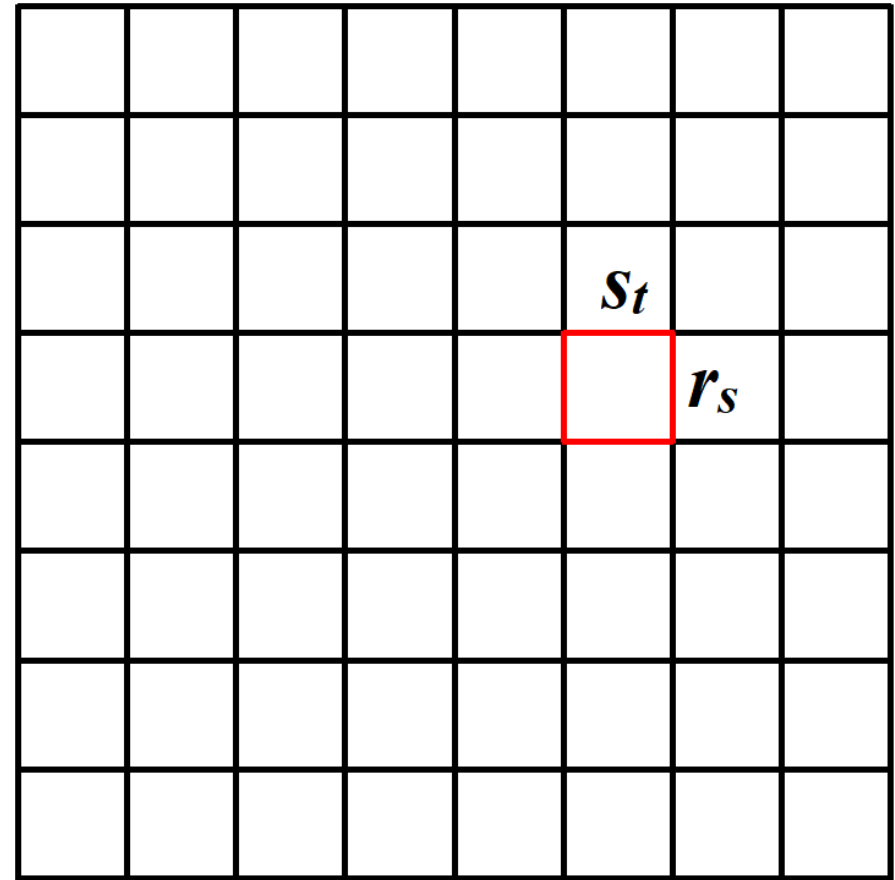
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# Connected $k$ -coverage Theory

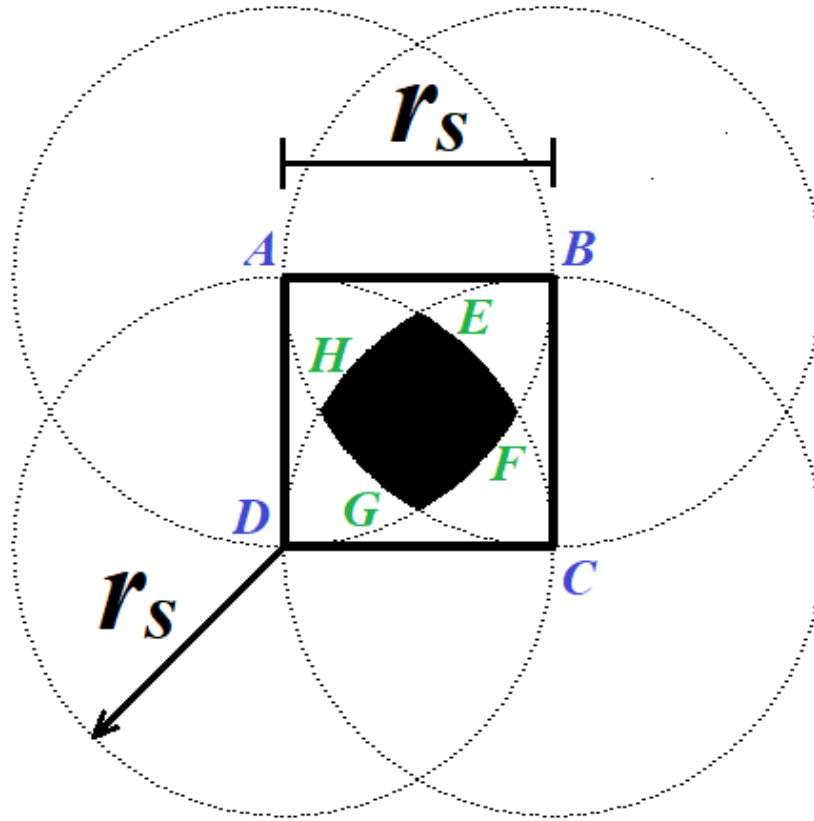
We solve this problem using Square tiles by following below steps,

1. Generate a Square Tessellation.
2. Construct Cusp Square areas.



$$S_t = r_s$$

# Cusp Square area of Square Tile

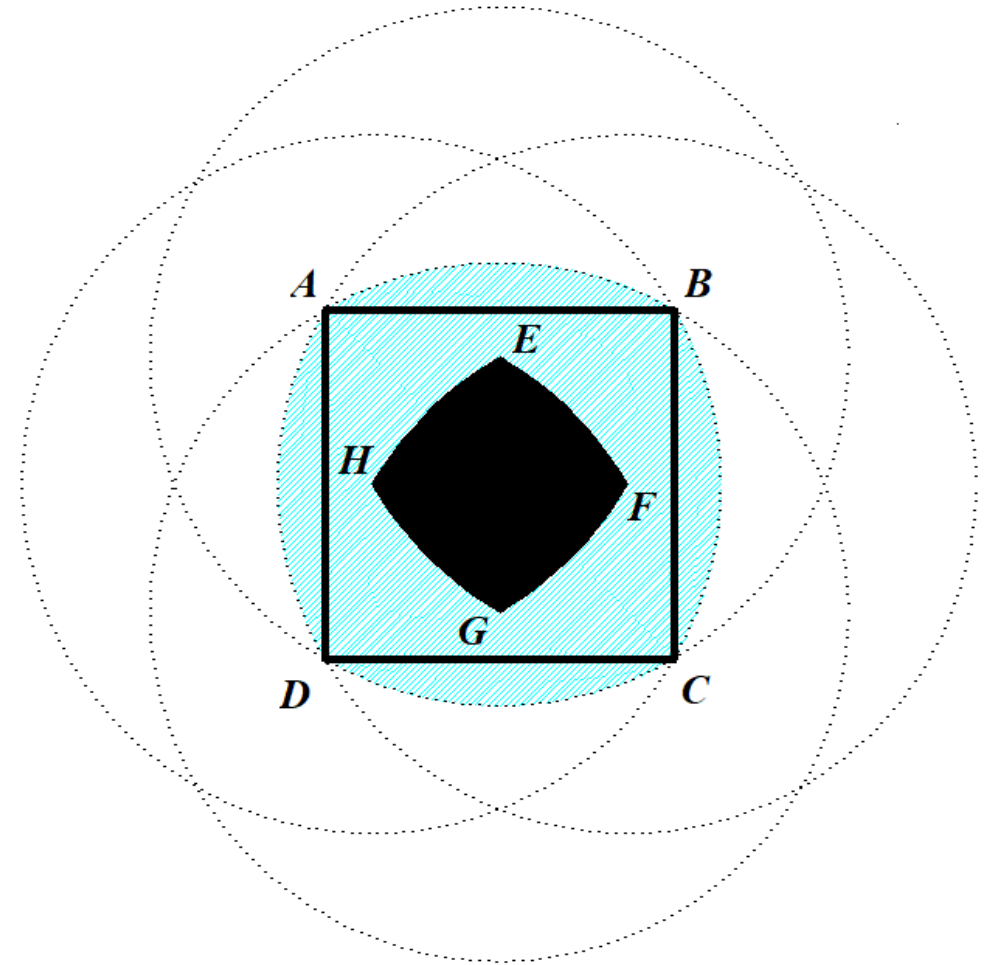


# $k$ -coverage Area of each Square Tile

$k$ -covered area  $A_k$  :

$$A_k = \left( \frac{2\pi + 3 - 3\sqrt{3}}{3} \right) r_s^2$$

$r_s$ : Sensing range of the sensors



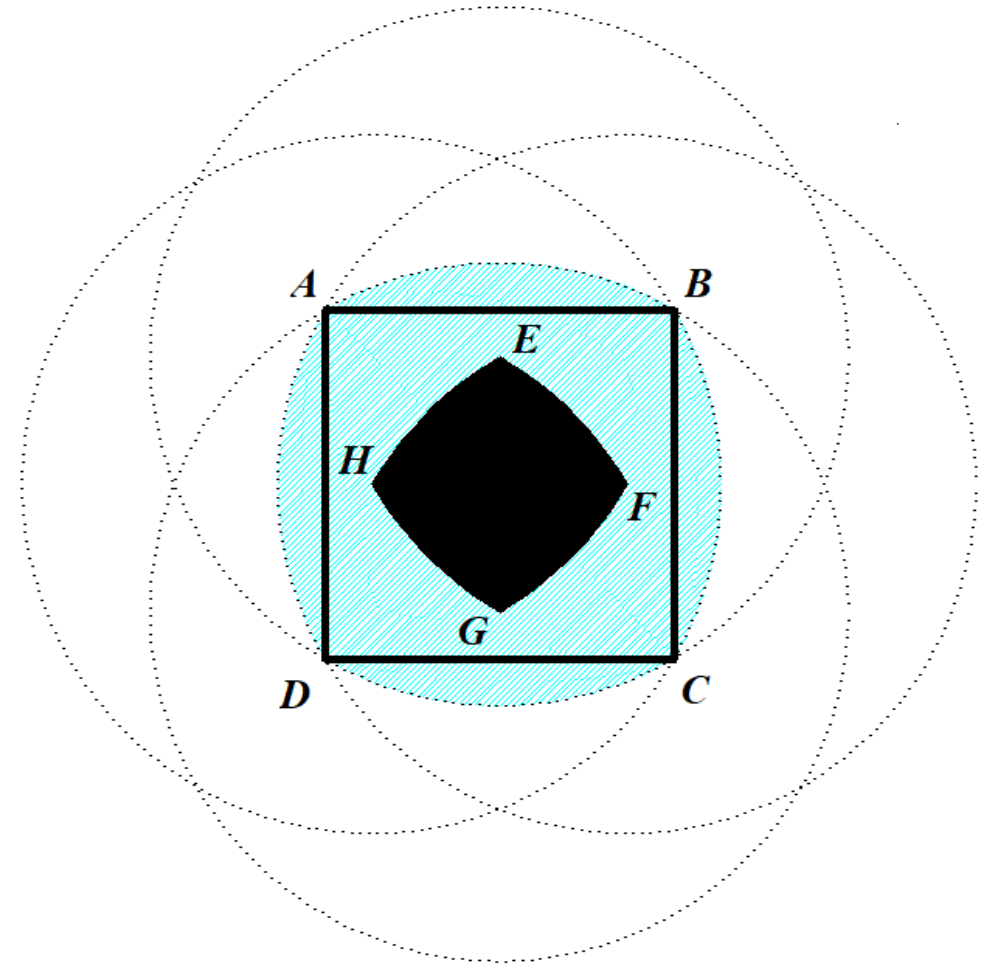


# Planar Sensor Density

$$\lambda(k, r_s) = \frac{0.734k}{r_s^2}$$

$r_s$  : Sensing range of the sensors

$$k \geq 1$$

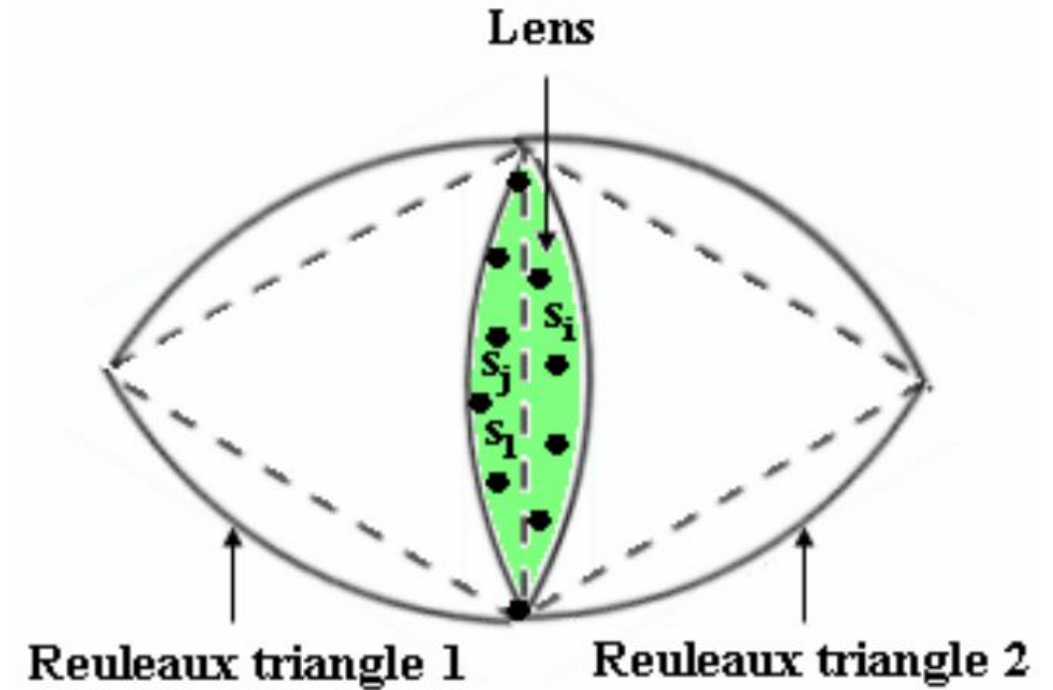


# Planar Sensor Density

$$\lambda(k, r_s) = \frac{0.814k}{r_s^2}$$

$r_s$  : Sensing range of the sensors

$$k \geq 1$$

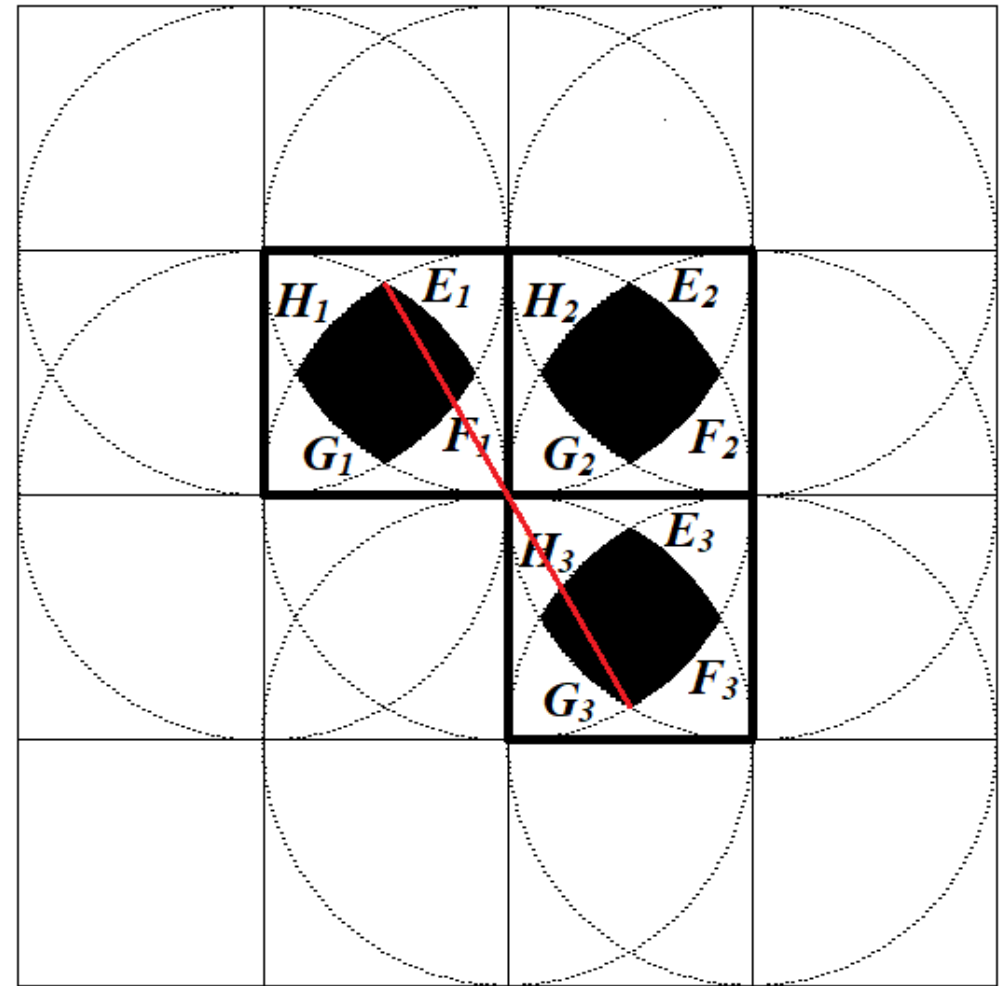


# Network connectivity relationship

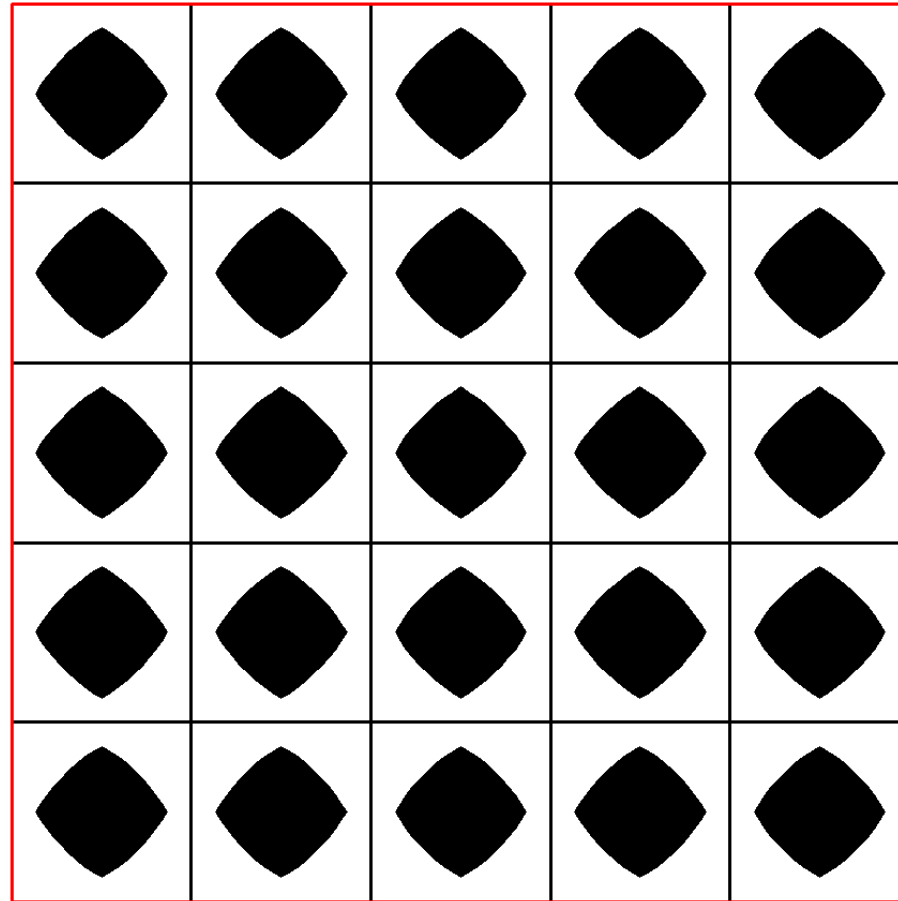
$$r_c \geq 2r_s$$

$r_s$  : Sensing range of the sensors

$r_c$  : Communication range of the sensors



# $\underline{k}$ -coverage using Cusp Squares ( $k$ -CSqu) Protocol



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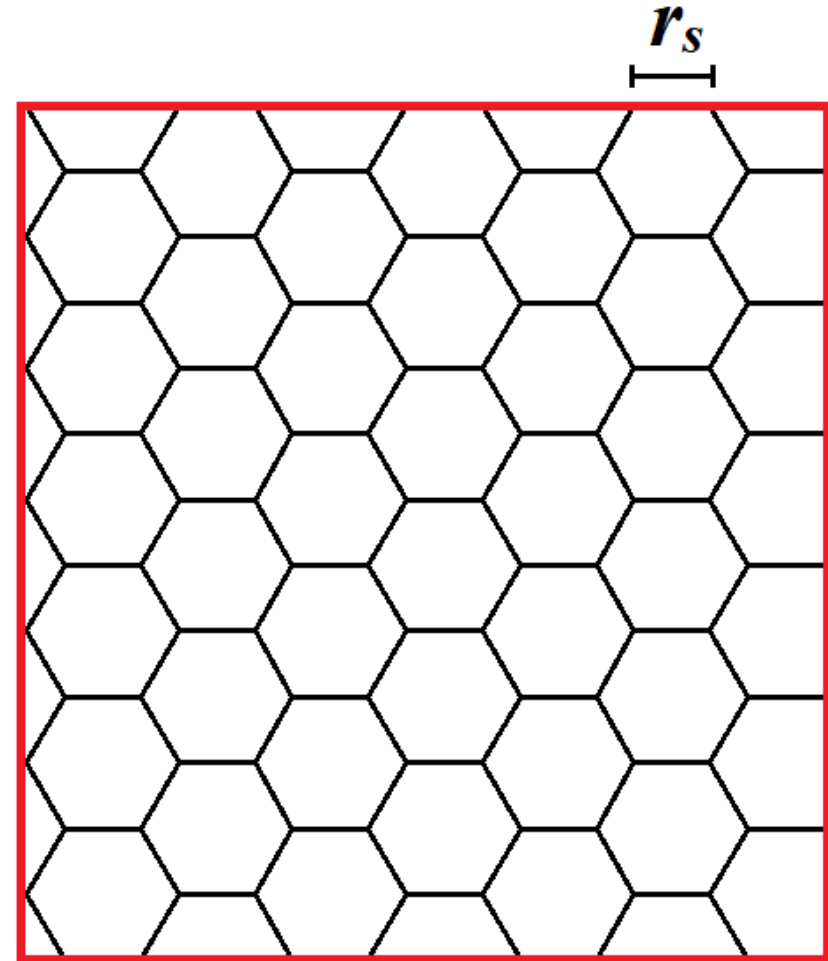
# Connected $k$ -coverage Theory

*Achieve 1-coverage of PWSN*

- Place sensors at centers of regular hexagonal tiles.

*Planar sensor density for 1-coverage:*

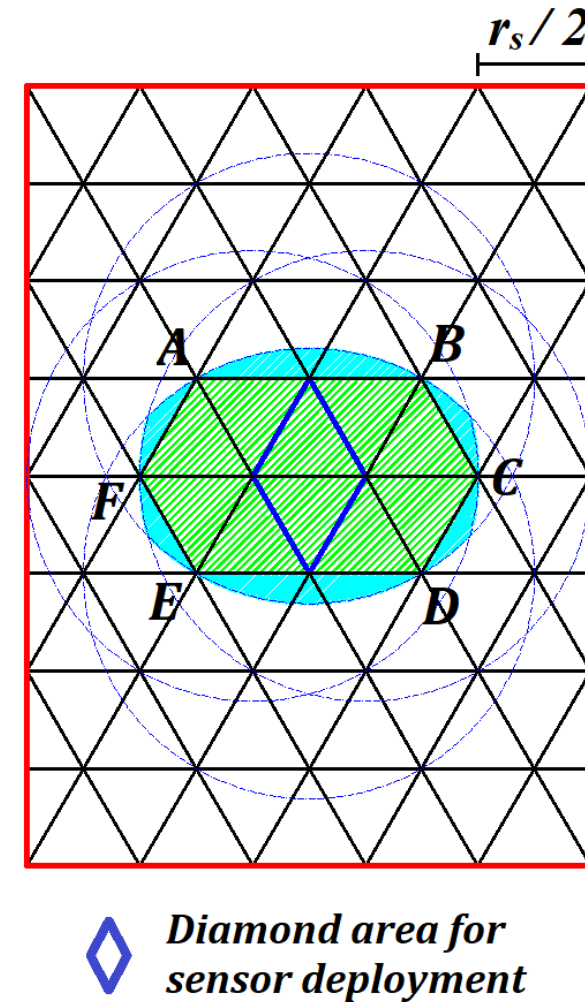
$$\lambda(r_s) = \frac{0.38}{r_s^2}$$



# Construction of Irregular Hexagonal Tile

Modify regular hexagon tessellation side length to  $r_s / 2$ .

Consider diamond area formed by two equilateral triangles of same base in the tessellation.



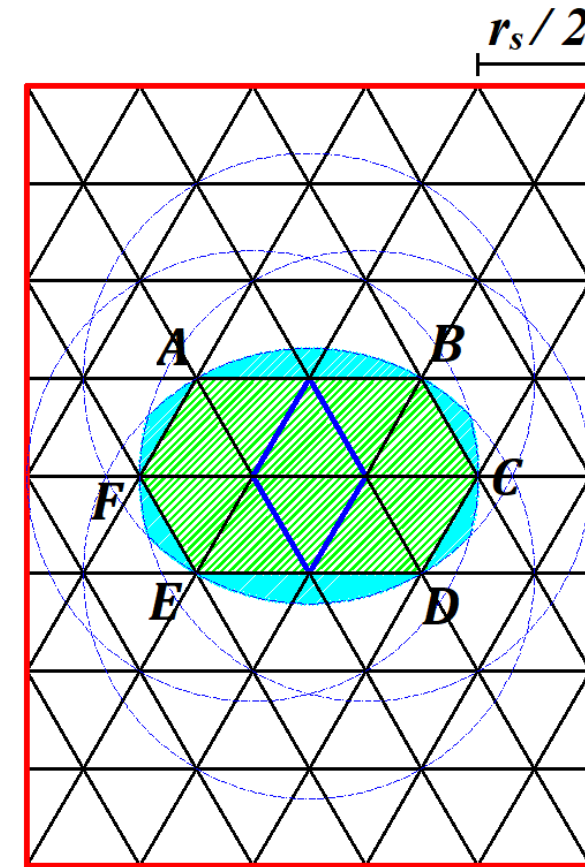
# $k$ -coverage area and Planar Sensor Density

$k$ -coverage area for the configuration is:

$$A_k = \left[ \pi + \frac{\sqrt{3}}{8} - \frac{\sqrt{15}}{4} - 4 \sin^{-1} \left( \frac{1}{4} \right) \right] r_s^2$$

Planar Sensor Density:

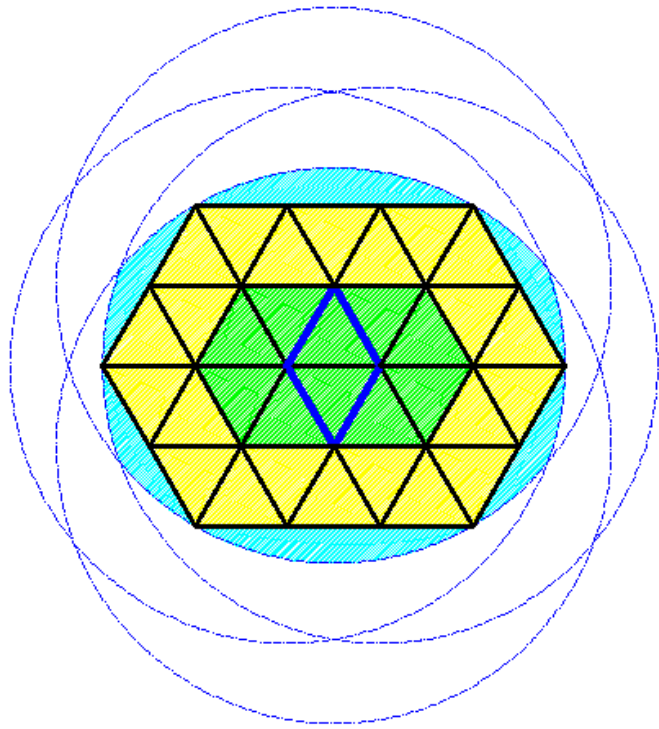
$$\lambda(k, r_s) = \frac{0.7251 k}{r_s^2}$$



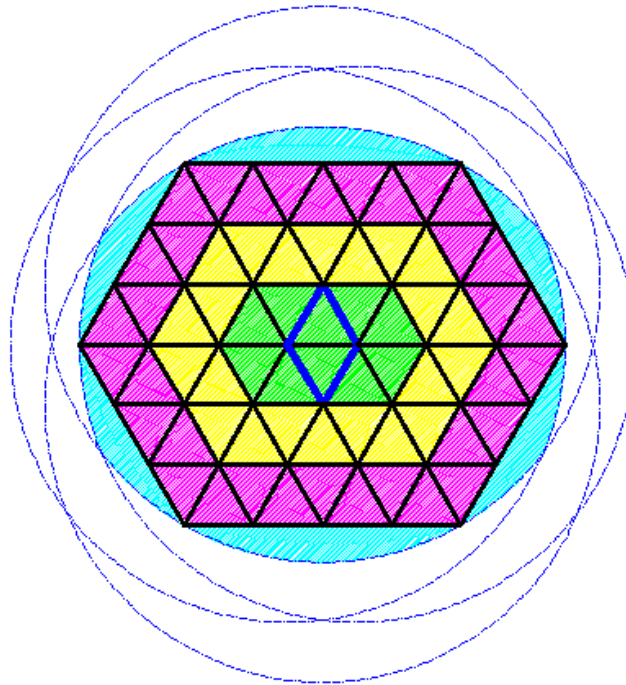
 *Diamond area for sensor deployment*



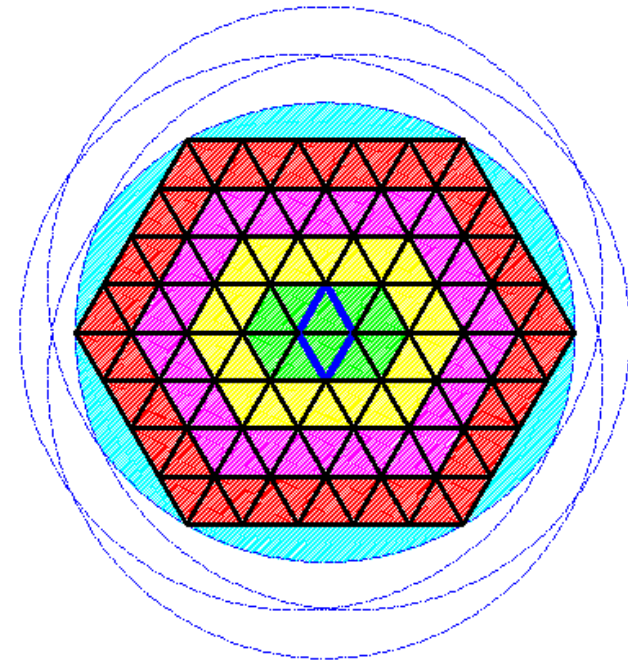
# Irregular Hexagonal Tiles for different values of $n$



(a)  $IrHx(r_s / 3)$



(b)  $IrHx(r_s / 4)$



(c)  $IrHx(r_s / 5)$

# Generalized Irregular Hexagonal Tile

## $IrHx(r_s/n)$

$\overline{AB}$	$\overline{BC}$	$\overline{CD}$	$\overline{DE}$	$\overline{EF}$	$\overline{FA}$	# Rings
$r_s$	$\frac{(n-1)r_s}{n}$	$\frac{(n-1)r_s}{n}$	$r_s$	$\frac{(n-1)r_s}{n}$	$\frac{(n-1)r_s}{n}$	$n-1$

# # Triangles in $IrHx(r_s/n)$

$n$	$\overline{AB}$	$\overline{BC}$	$\overline{CD}$	$\overline{DE}$	$\overline{EF}$	$\overline{FA}$	# Rings
2	$r_s$	$r_s/2$	$r_s/2$	$r_s$	$r_s/2$	$r_s/2$	1
3	$r_s$	$2r_s/3$	$2r_s/3$	$r_s$	$2r_s/3$	$2r_s/3$	2
4	$r_s$	$3r_s/4$	$3r_s/4$	$r_s$	$3r_s/4$	$3r_s/4$	3
5	$r_s$	$4r_s/5$	$4r_s/5$	$r_s$	$4r_s/5$	$4r_s/5$	4

n	Ring #1	Ring #2	Ring #3	Ring #4
2	10			
3	10	22		
4	10	22	34	
5	10	22	34	46

Total number of equilateral triangles  $N$ ,  $IrHx(r_s/n)$  is given by:

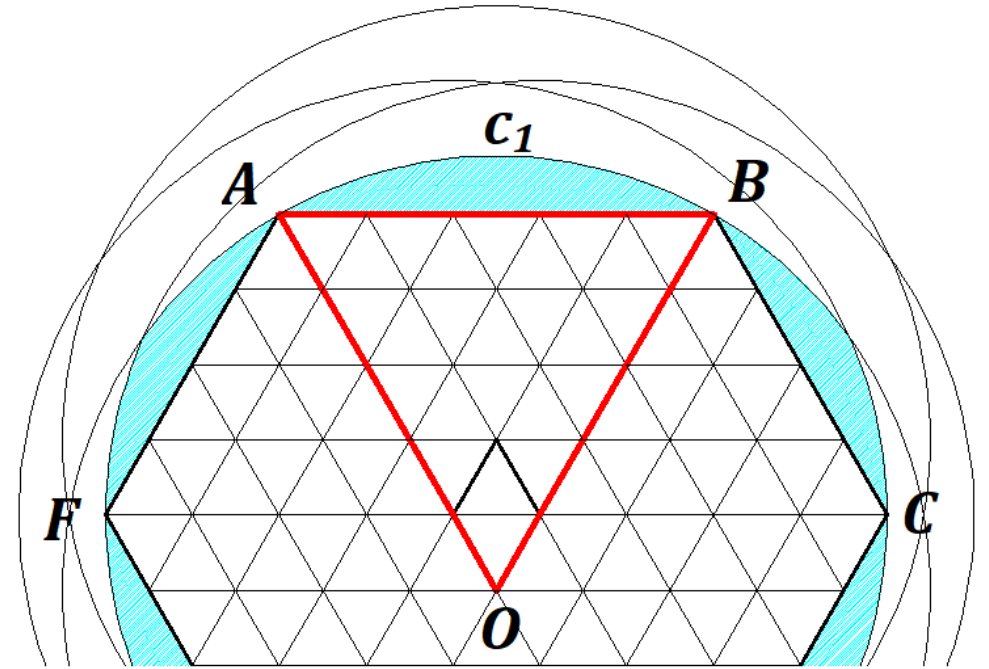
$$N = 2(n - 1)(3n - 1)$$

$$n \geq 1$$

$k$ -coverage Area over the  
Longer side of  $IrHx(r_s/n)$

$$A_{LS} = \left( \frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) r_s^2$$

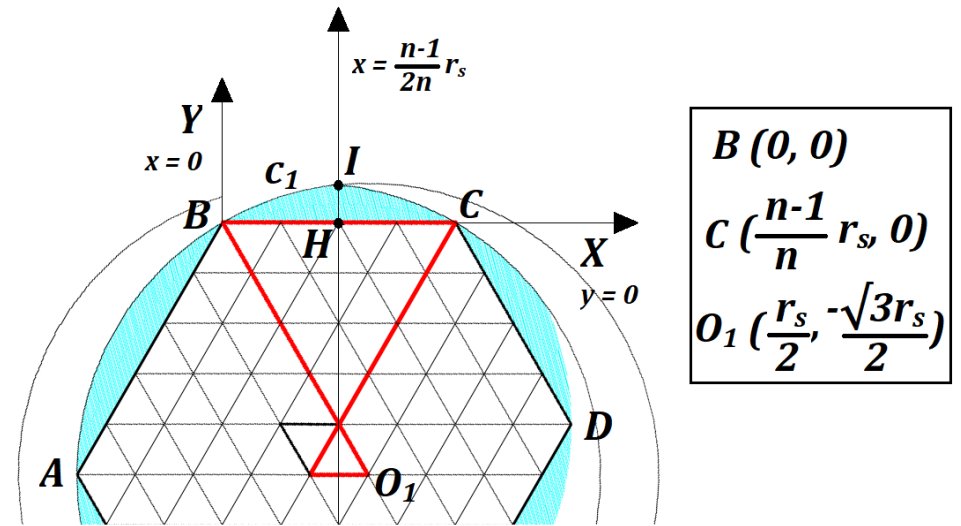
$r_s$ : Sensing range of sensor



# $k$ -coverage Area over the Shorter side of $IrHx(r_s/n)$

$$A_{SS} = \left[ \frac{\pi}{6} + \sin^{-1} \left( \frac{-1}{2n} \right) - \frac{\sqrt{4n^2 - 1}}{4n^2} - \frac{\sqrt{3}(n - 2)}{4n} \right] r_s$$

$r_s$  : Sensing range of sensor



# $k$ -coverage area of $IrHx(r_s/n)$ Tile

The  $k$ -covered area  $A_k$ :

$$A_k = \left[ \pi + \frac{(3n^2 - 6n + 2)\sqrt{3}}{4n^2} - \frac{\sqrt{4n^2 - 1}}{n^2} - 4 \sin^{-1} \left( \frac{1}{2n} \right) \right] r_s^2$$

$r_s$ : Sensing range of sensor

$$n \geq 1$$

# Planar Sensor Density of $IrHx(r_s/n)$ Tile

The planar sensor density  $\lambda(k, r_s, n)$  :

$$\lambda(k, r_s, n) = \frac{k}{\left[ \pi + \frac{(3n^2 - 6n + 2)\sqrt{3}}{4n^2} - \frac{\sqrt{4n^2 - 1}}{n^2} - 4 \sin^{-1} \left( \frac{1}{2n} \right) \right] r_s^2}$$

$r_s$ : Sensing range of sensor

$k \geq 1$  and  $n \geq 1$

# Planar Sensor Density of $IrHx(r_s/n)$ for different values of $n$

n	2	3	4	5	10	20	100	$\infty$
$\lambda(k, r_s, n)$	$\frac{0.7251 k}{r_s^2}$	$\frac{0.4267 k}{r_s^2}$	$\frac{0.3511 k}{r_s^2}$	$\frac{0.3168 k}{r_s^2}$	$\frac{0.2639 k}{r_s^2}$	$\frac{0.2431 k}{r_s^2}$	$\frac{0.2252 k}{r_s^2}$	$\frac{0.2252 k}{r_s^2}$

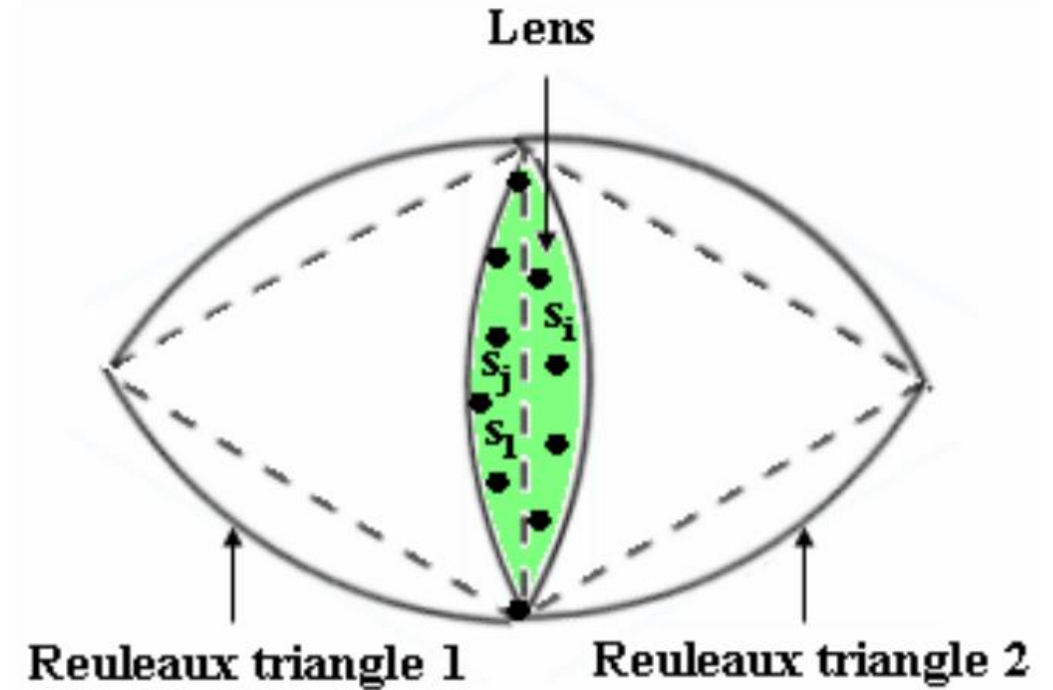


# Planar Sensor Density

$$\lambda(k, r_s) = \frac{0.814k}{r_s^2}$$

$r_s$  : Sensing range of the sensors

$$k \geq 1$$

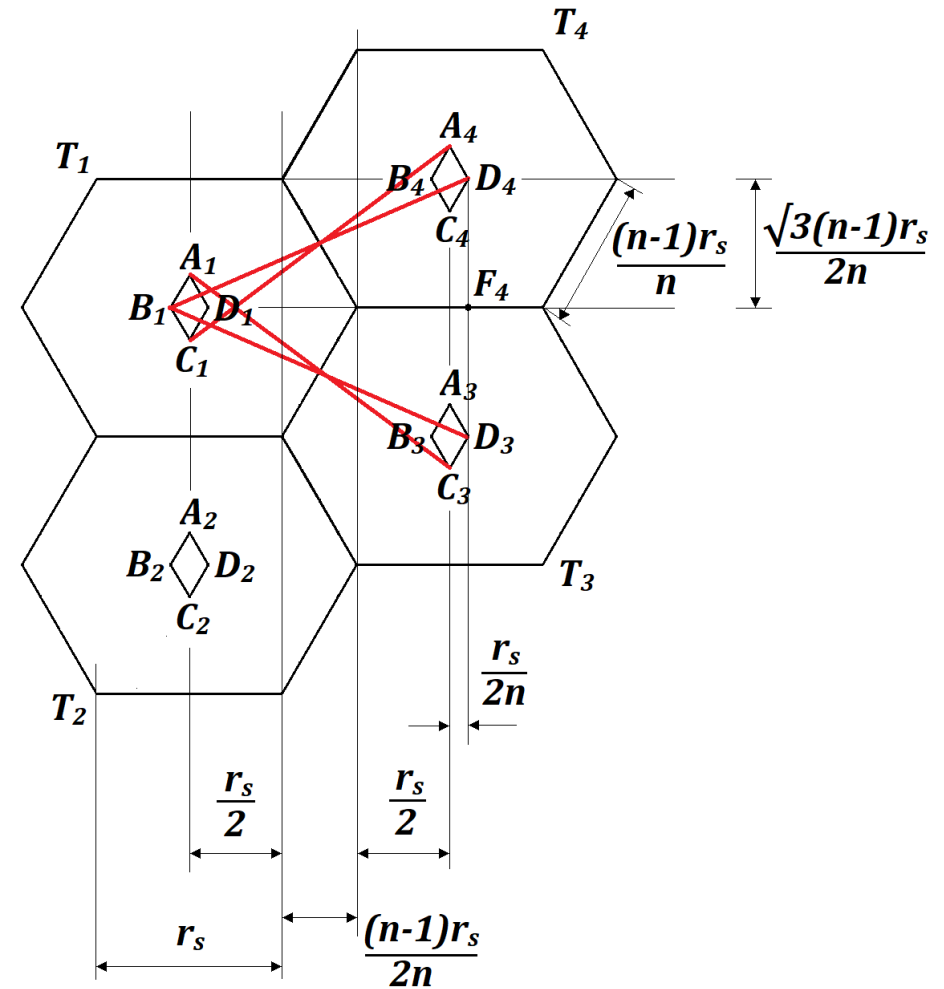


# Network connectivity relation

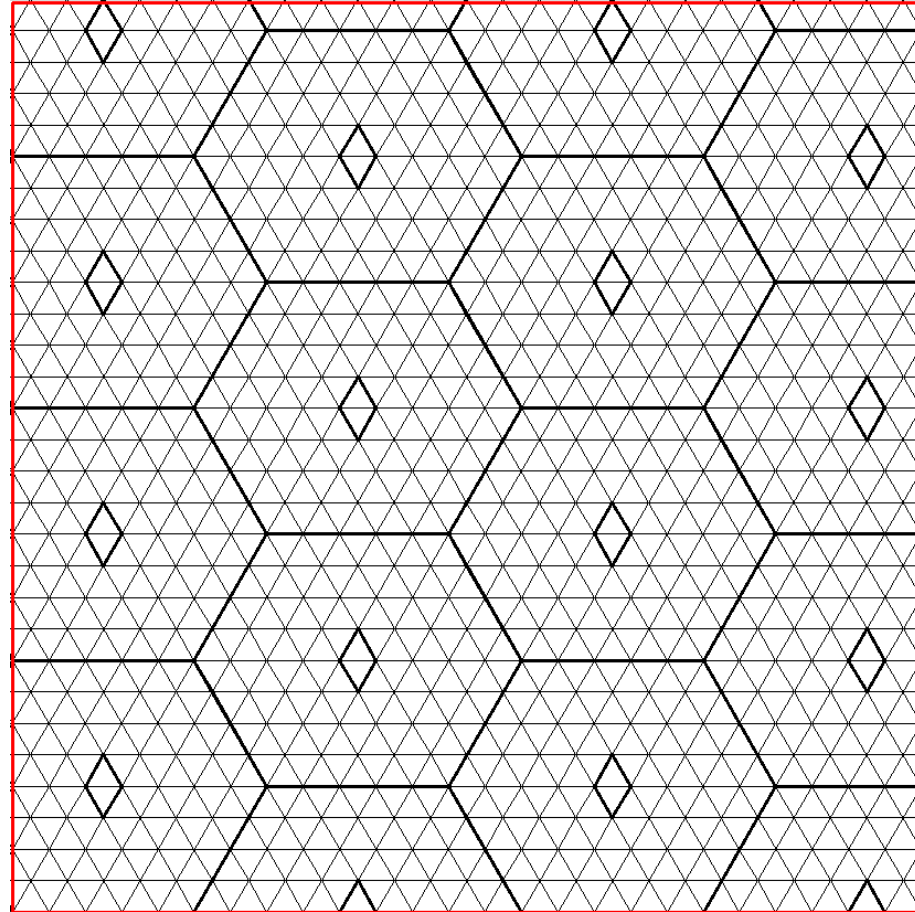
$$r_c \geq \frac{\sqrt{3n^2 + 1}}{n} r_s$$

$r_s$ : Sensing range of the sensors

$r_c$ : Communication range of the sensors



# $\underline{k}$ -coverage using Inner Diamonds ( $k$ -InDi) Protocol

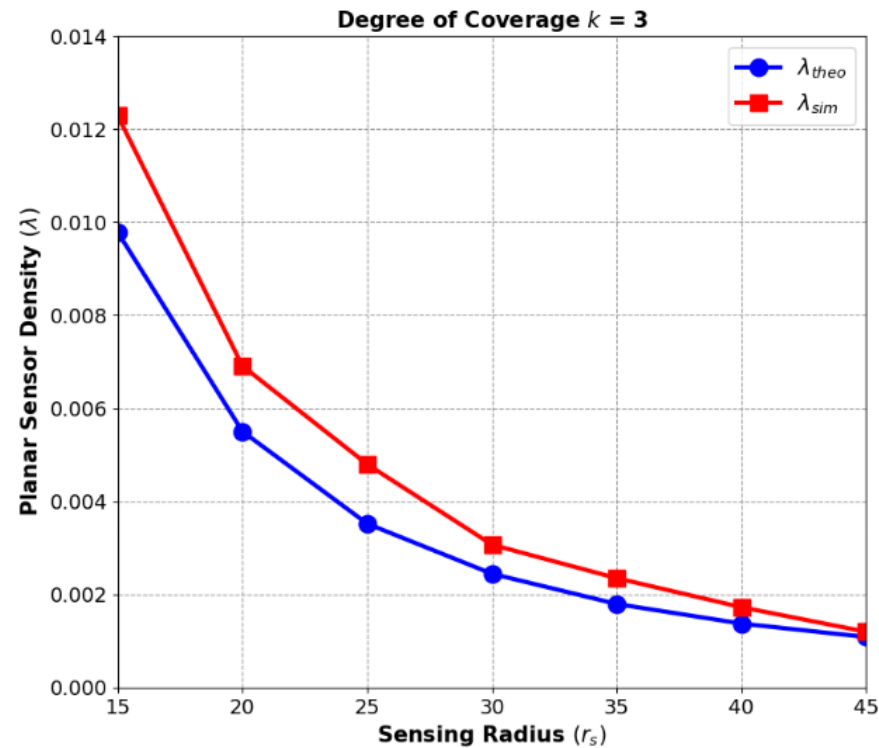


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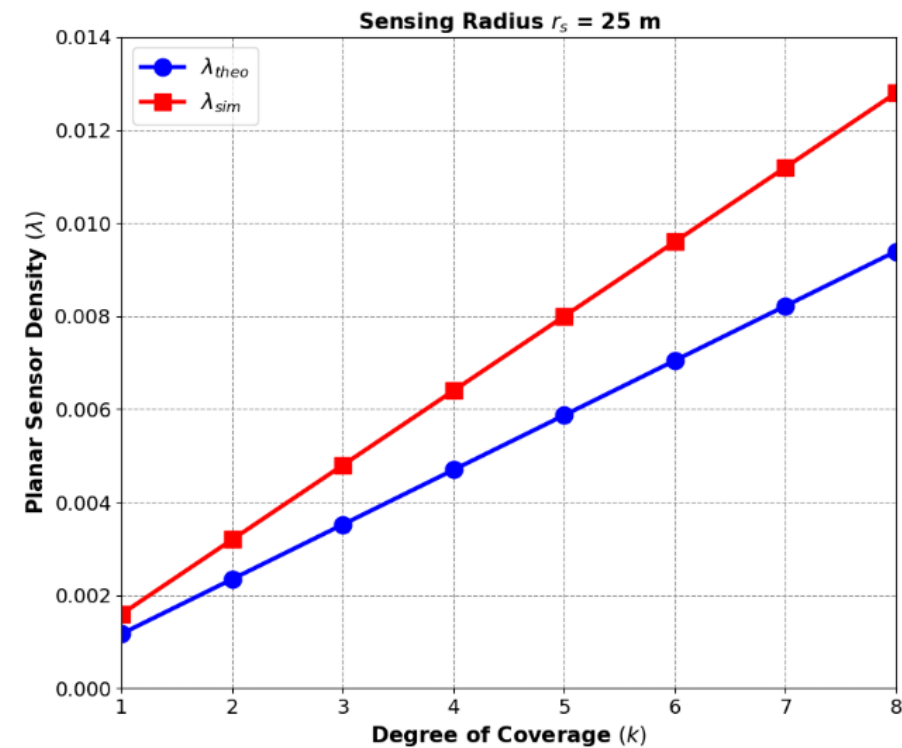
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# $k$ -CSqu Results

Planar sensor density  $\lambda$  vs. Sensing radius  $r_s$

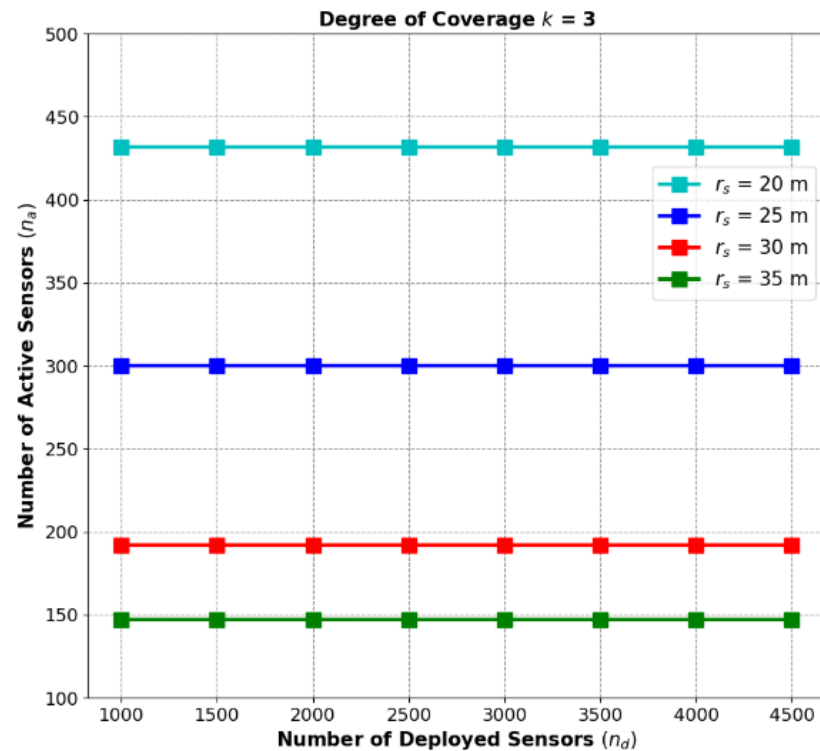


Planar sensor density  $\lambda$  vs. Degree of coverage  $k$

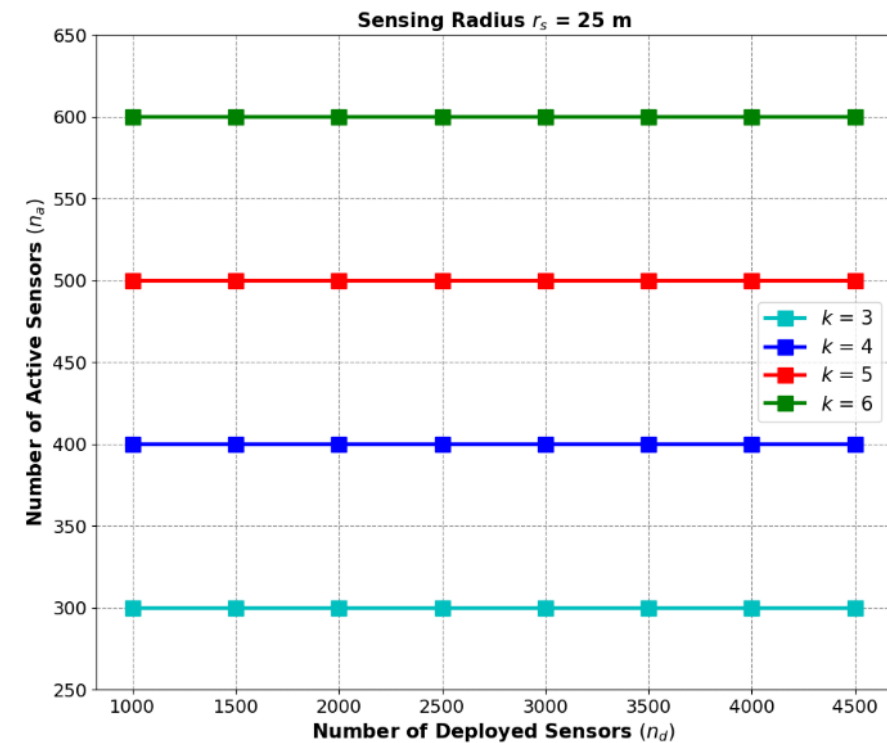


# $k$ -CSqu Results

Number of active sensors  $n_a$  vs. Number of deployed sensors  $n_d$  for different Sensing radius  $r_s$

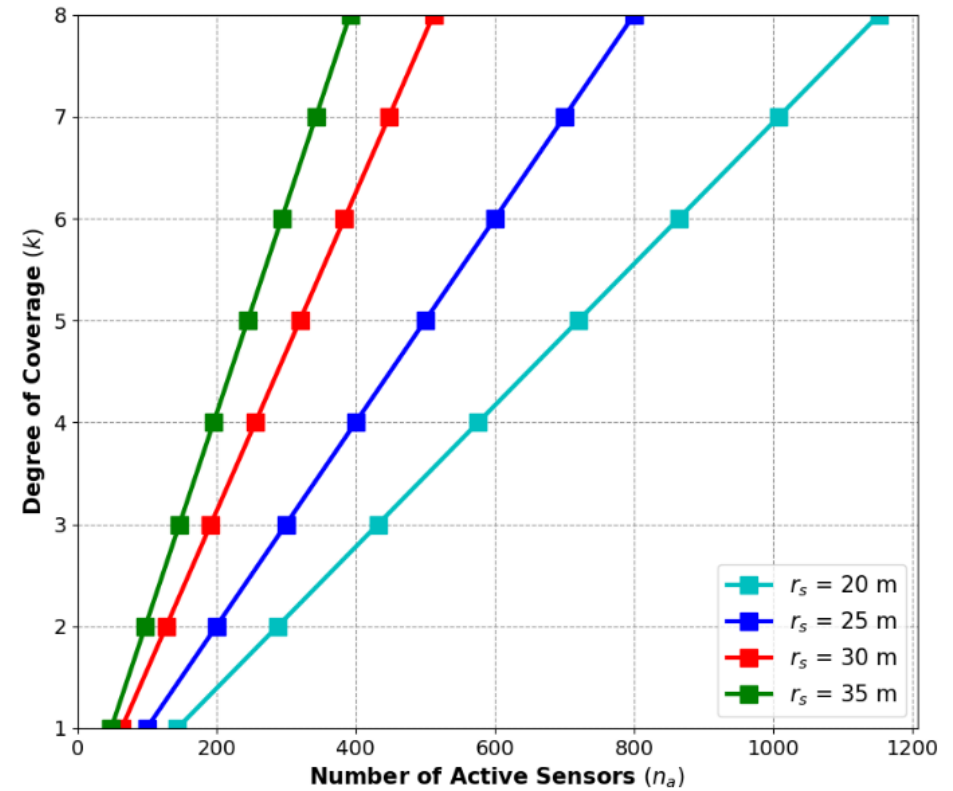


Number of active sensors  $n_a$  vs. Number of deployed sensors  $n_d$  for different Degree of coverage  $k$



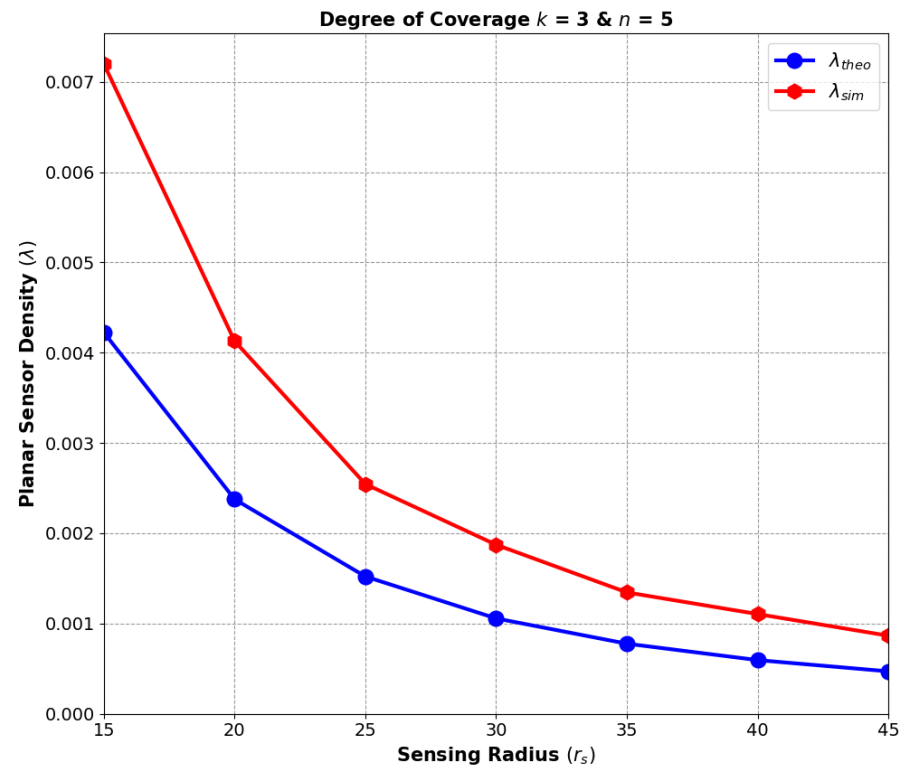
# $k$ -CSqu Results

Degree of coverage  $k$  versus Number of active sensor  $n_a$

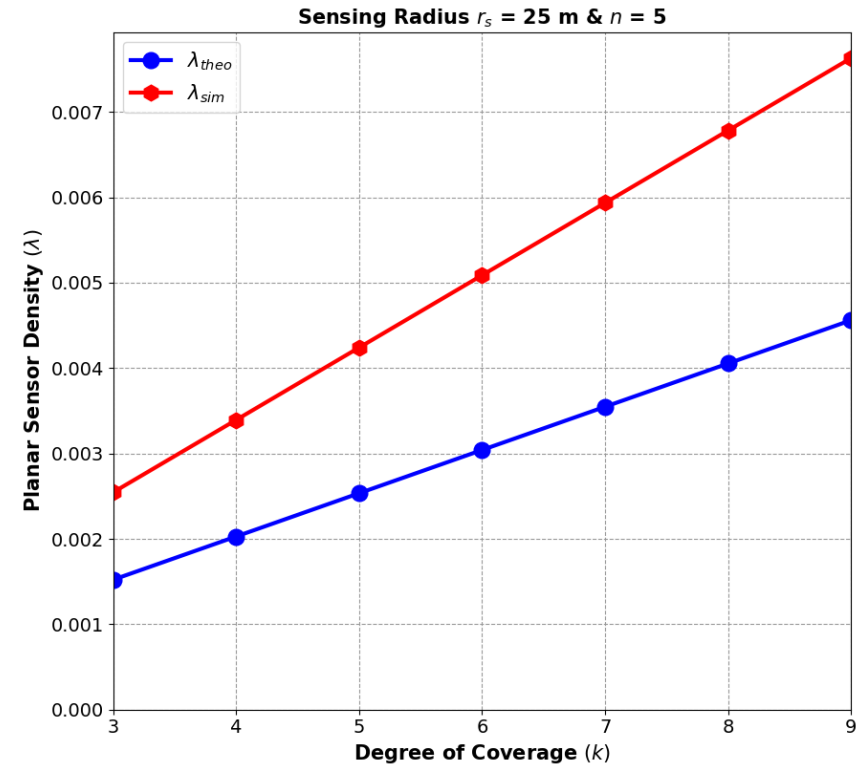


# $k$ -InDi Results

Planar sensor density  $\lambda$  vs. Sensing radius  $r_s$



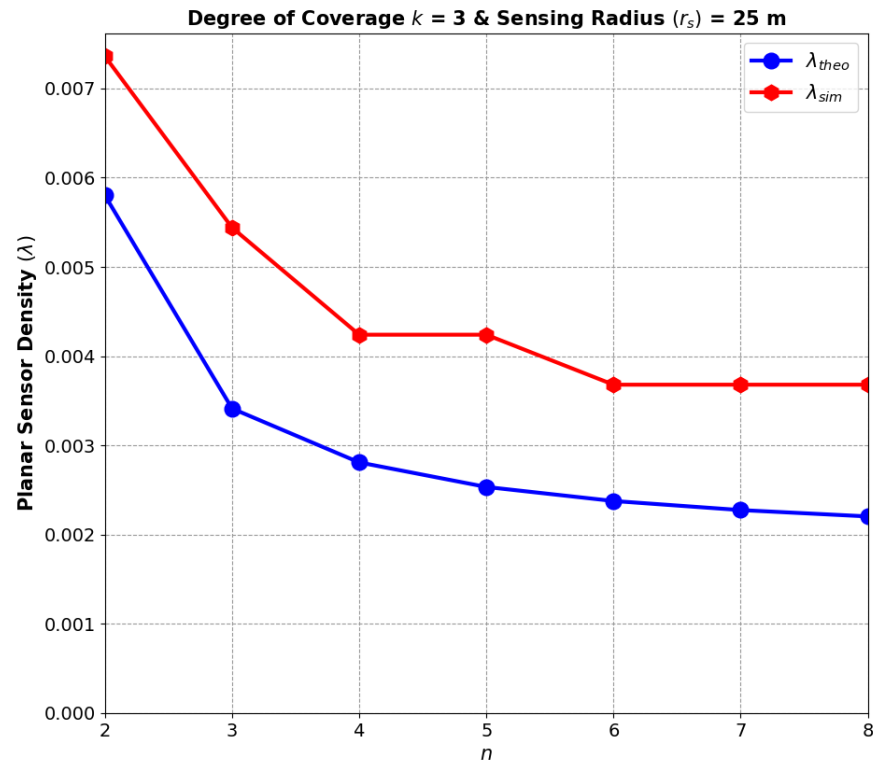
Planar sensor density  $\lambda$  vs. Degree of coverage  $k$



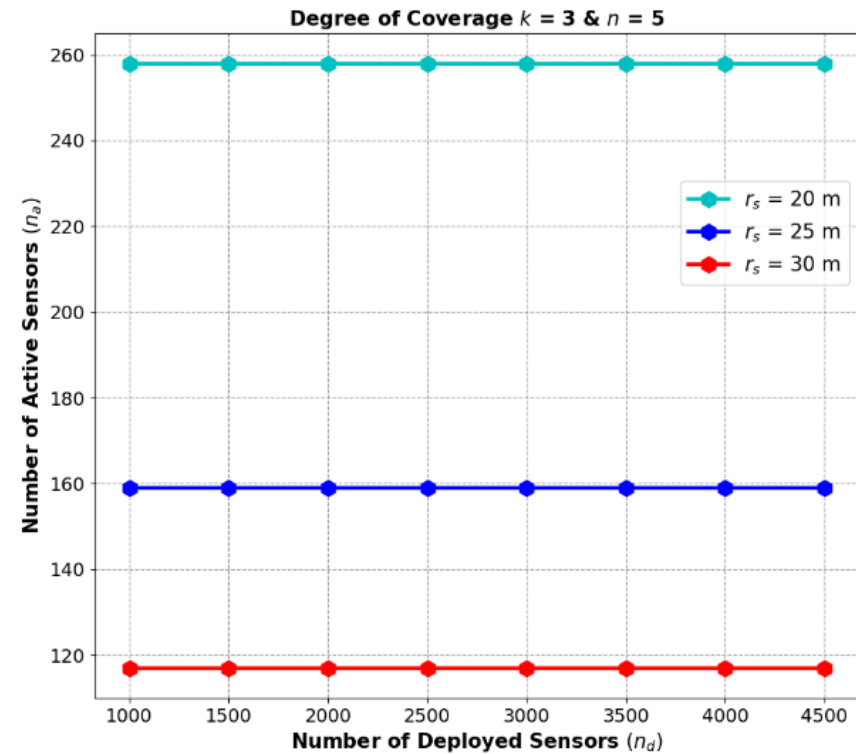


# $k$ -InDi Results

Planar sensor density  $\lambda$  vs. factor  $n$

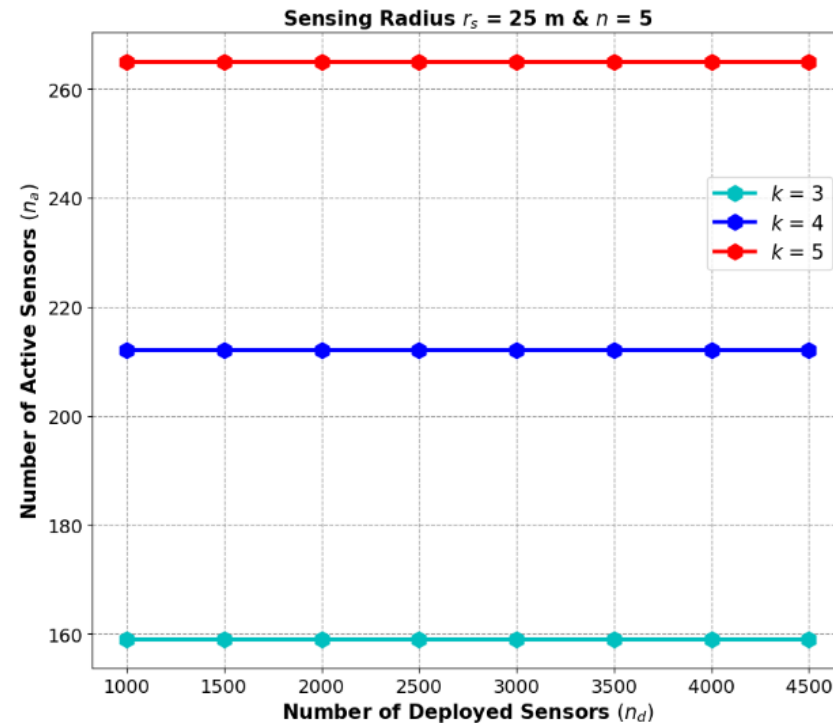


Number of active sensors  $n_a$  vs. Number of deployed sensors  $n_d$  for different Sensing radius  $r_s$

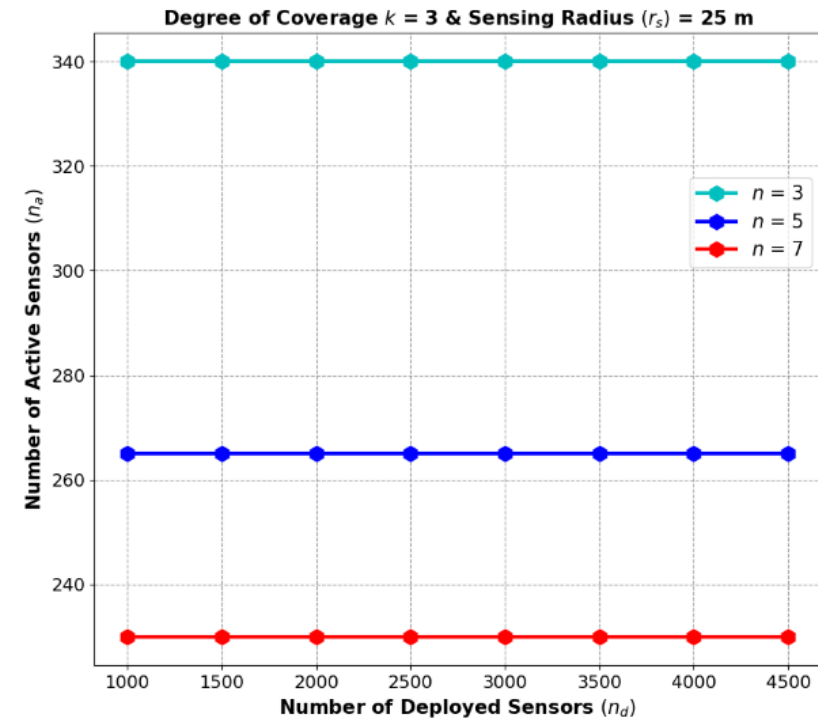


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Number of active sensors  $n_a$  vs. Number of deployed sensors  $n_d$  for different Degree of coverage  $k$

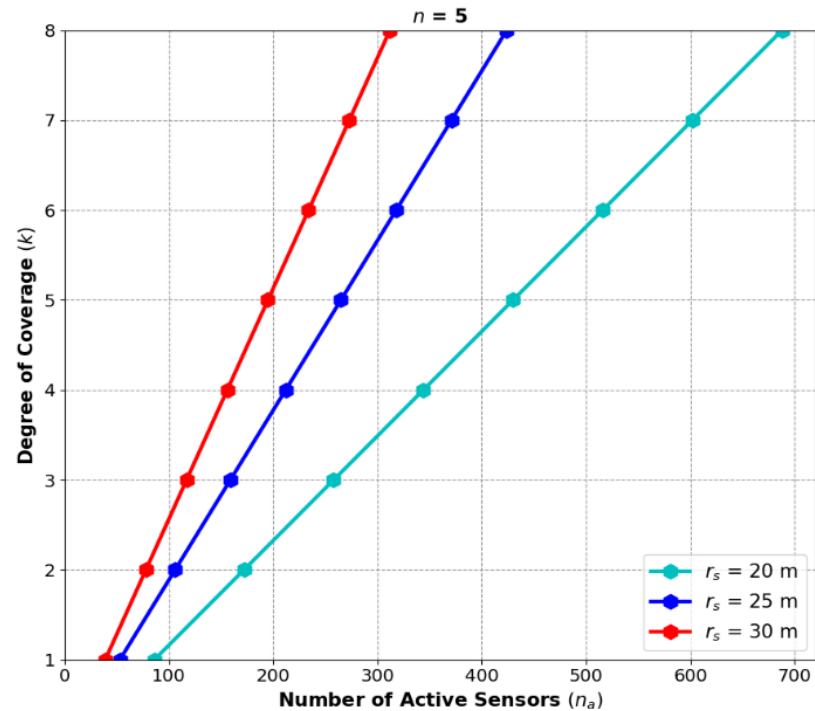


Number of active sensors  $n_a$  vs. Number of deployed sensors  $n_d$  for different factor  $n$

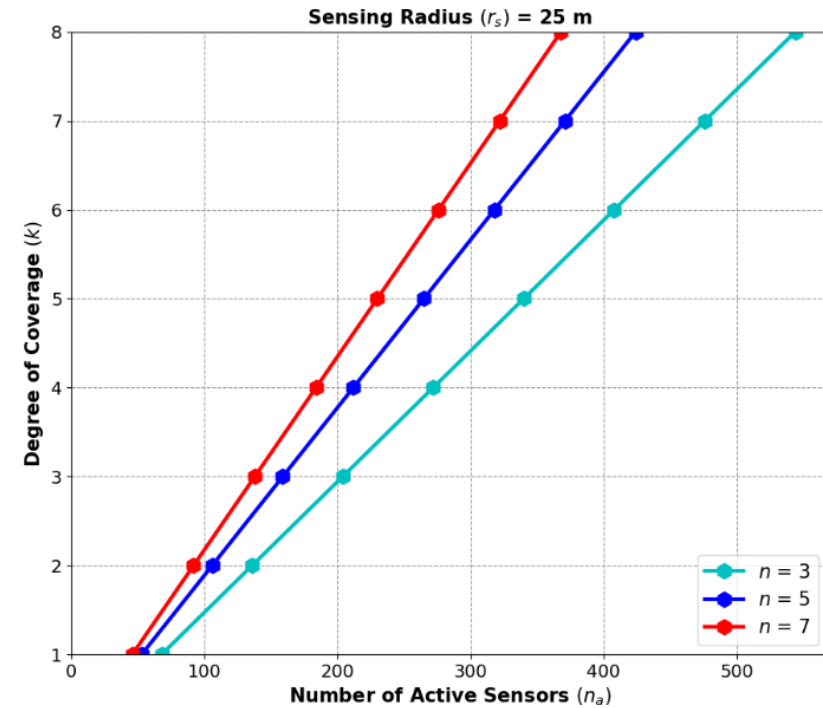


# $k$ -InDi Results

Degree of coverage  $k$  versus Number of active sensor  $n_a$  for different Sensing radius  $r_s$

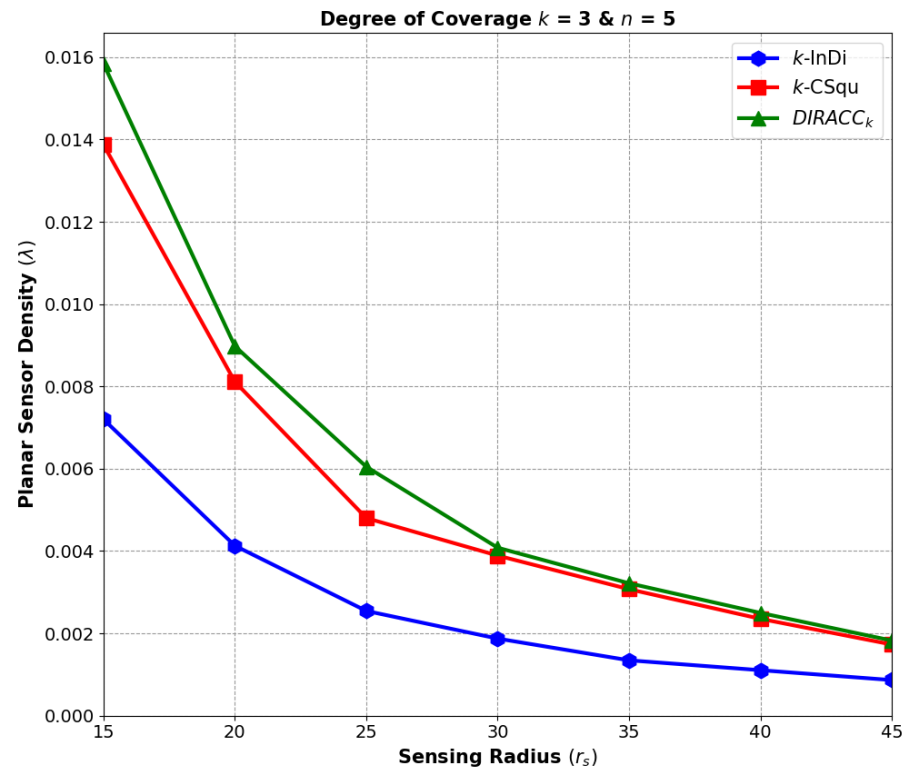


Degree of coverage  $k$  versus Number of active sensor  $n_a$  for different factor  $n$

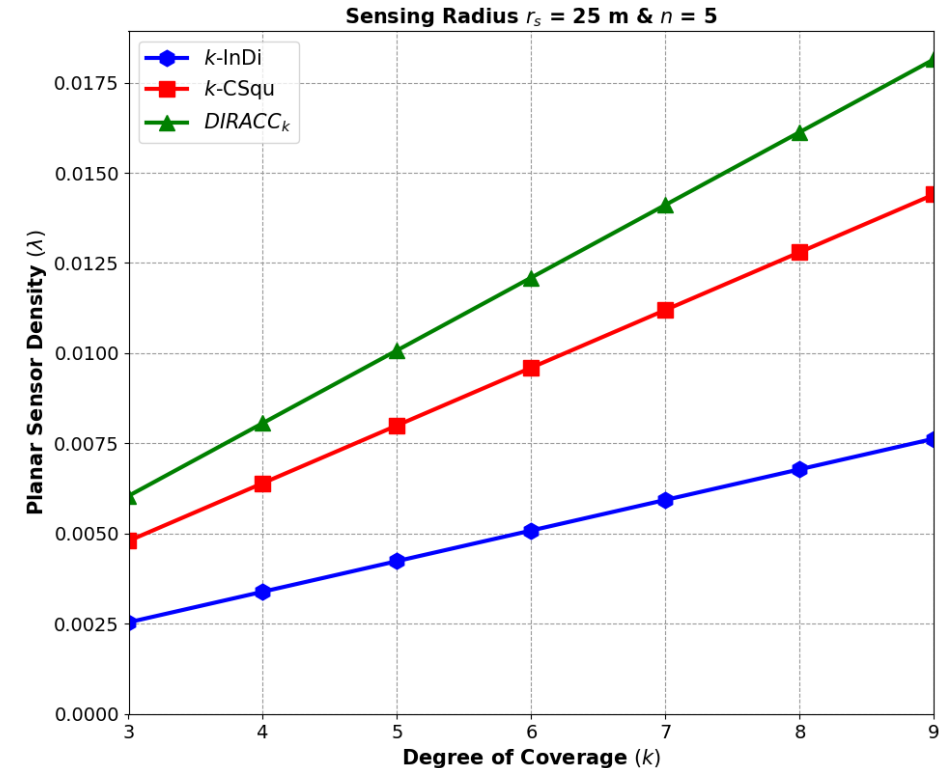


# Comparison of $k$ -CSqu and $k$ -InDi with $\text{DIRACC}_k$

Planar sensor density  $\lambda$  vs. Sensing radius  $r_s$

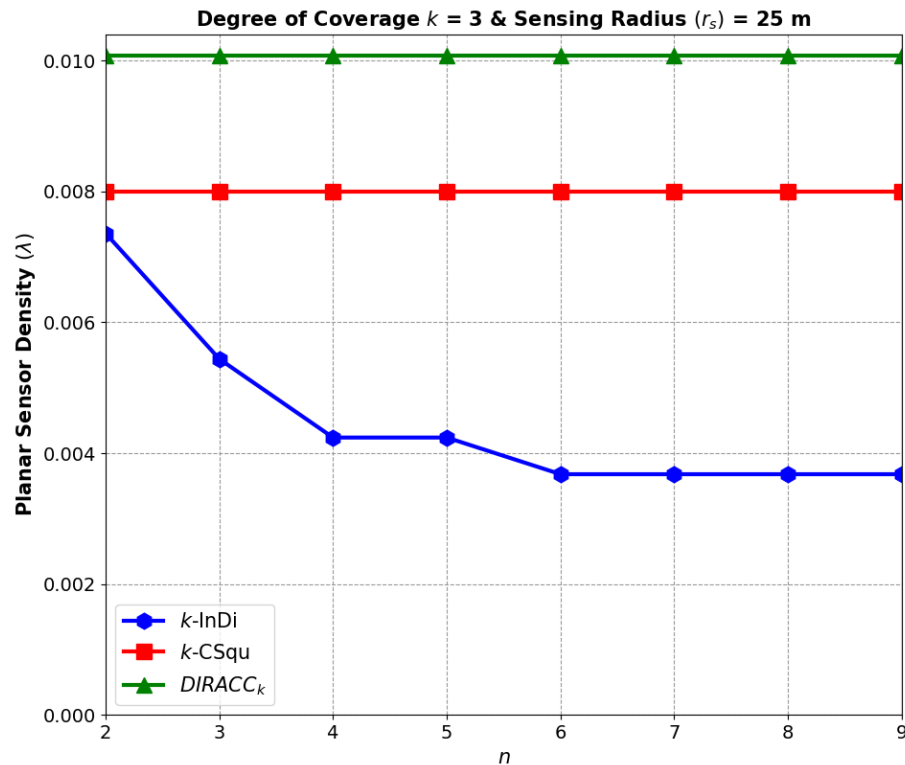


Planar sensor density  $\lambda$  vs. Degree of coverage  $k$

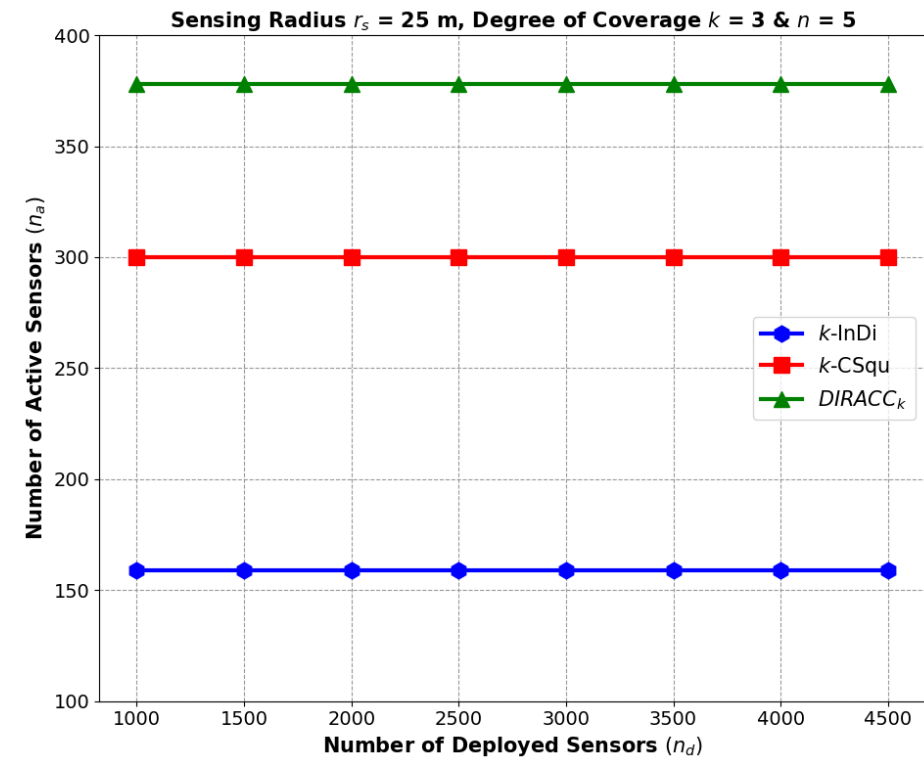


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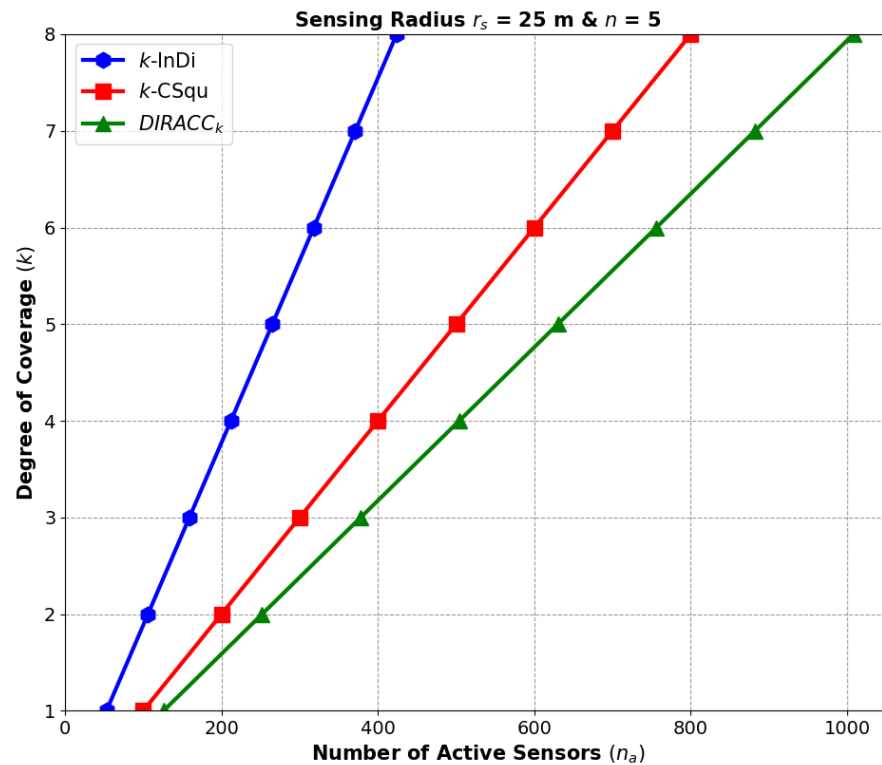


Number of active sensors  $n_a$  versus Number of deployed sensors  $n_d$

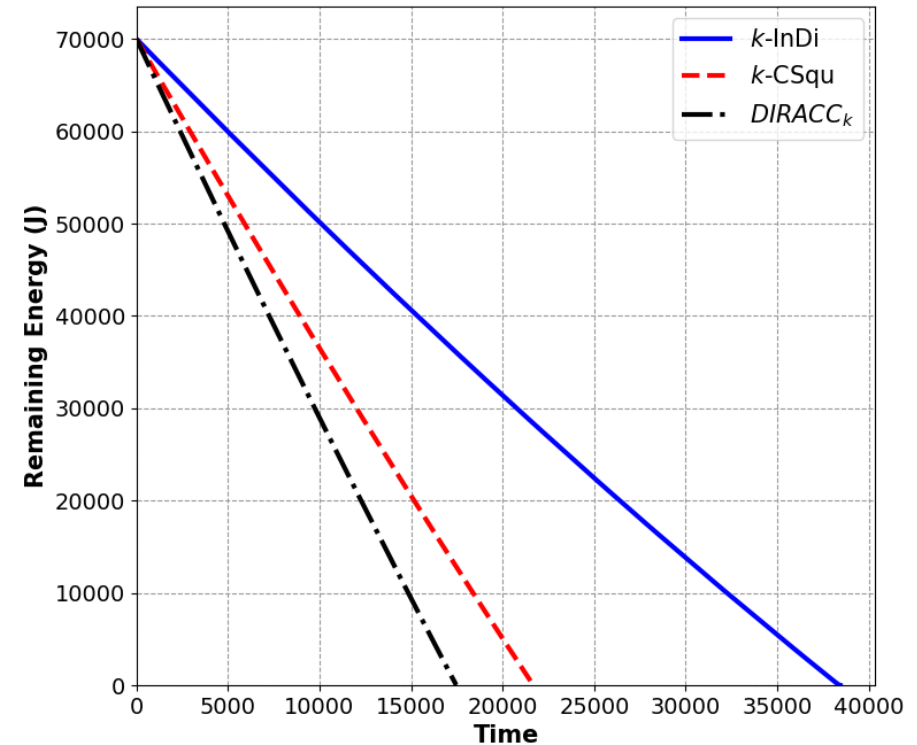


# Comparison of $k$ -CSqu and $k$ -InDi with $\text{DIRACC}_k$

Degree of coverage  $k$  versus Number of active sensor  $n_a$

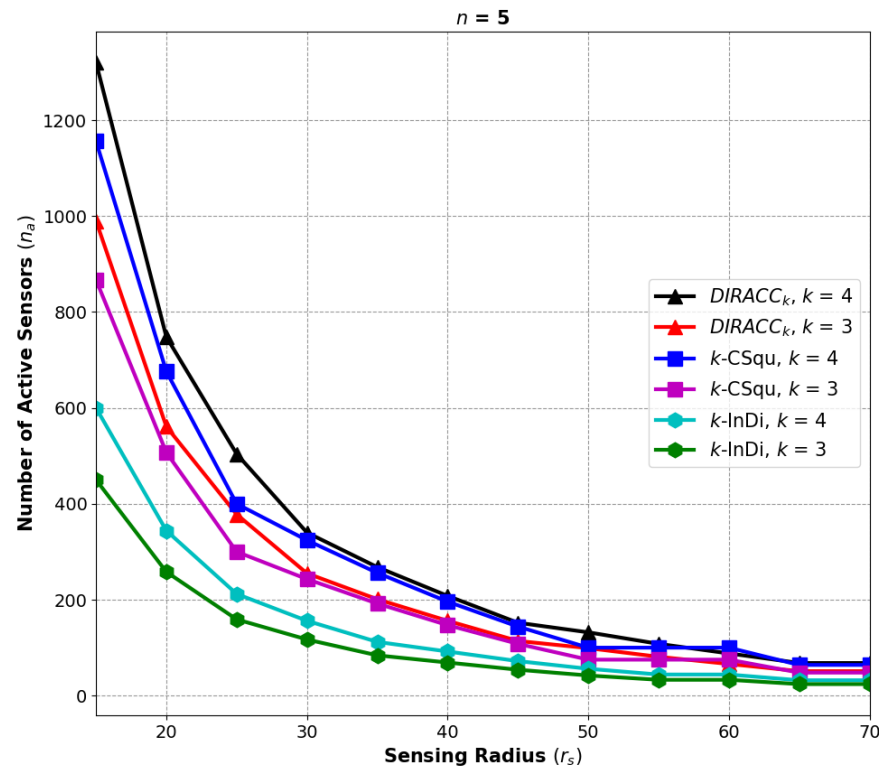


Remaining Energy vs. Time

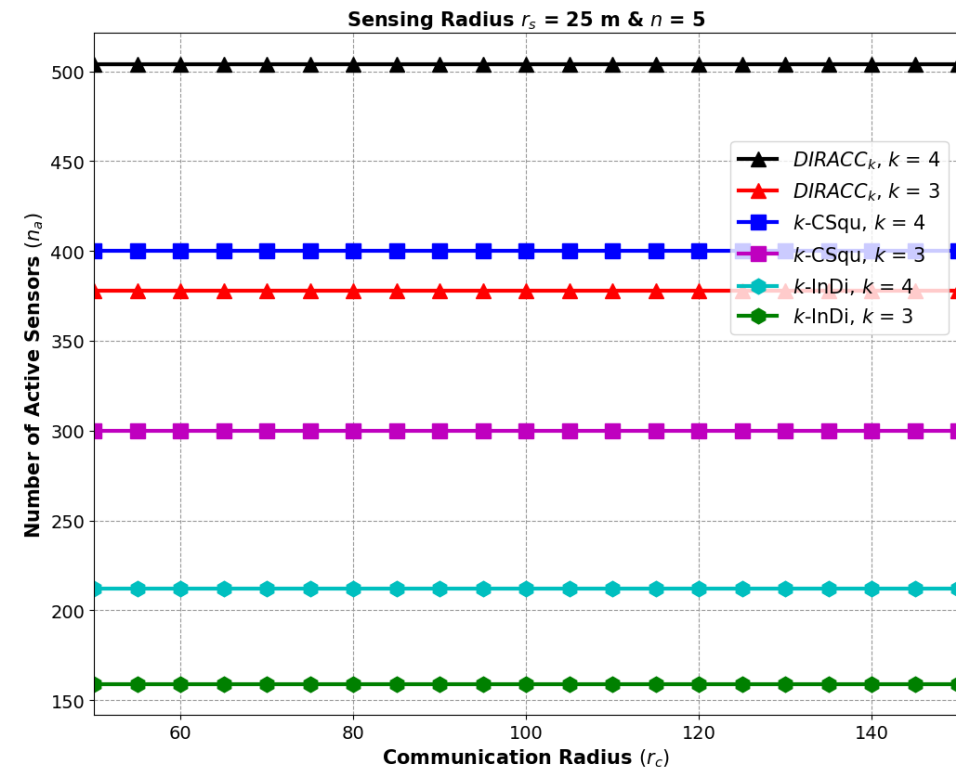


# Comparison of $k$ -CSqu and $k$ -InDi with $\text{DIRACC}_k$

Number of active sensors  $n_a$  vs. Sensing radius  $r_s$



Number of active sensors  $n_a$  vs. Communication radius  $r_c$



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# Summary of Research

- Investigated connected  $k$ -coverage problem of PWSNs.
- Addressed sensor placement issue for both tessellations.
- Calculated planar sensor density for  $k$ -coverage for both tessellations.
- Established network connectivity relation for both tessellations.
- Proposed centralized  $k$ -coverage protocols,  $k$ -CSqu and  $k$ -InDi.

# Future scope of Research

- Find optimal value of  $n$ .
- Extend our  $k$ -CSqu and  $k$ -InDi approaches to heterogeneous sensors.
- Extend our  $k$ -CSqu and  $k$ -InDi approaches to stochastic sensing model.
- Extend our both theories to three-dimensional space.

*Thank You !!!*